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SPORT TOURMANENTS WITH MINIMUM NUMBER OF TRAVELING FOR FIVE AND SIX TEAMS

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Abstract

There are many sport tournaments nowadays. One type of sport traveling tournament could be considered a traveling tournament problem (TTP) or double round robin since each team has to plays against another team twice: one home game and one opponents home game. If there are n teams and each team is required to plays against every other team in the first $n-1$ games, this is called a half traveling tournament problem (HTTP). If the HTTP also requires that the last $n-1$ games are ordered exactly like the first $n-1$ games with reversed venues, then it is called a mirrored traveling tournament problem (MTTP). This research aims to study about how to schedule the sport tournament with minimum total number of traveling of all teams in case of five and six teams. The proofs and examples of tournaments with minimum total number of traveling are presented. The result shows that the minimum total number of all team traveling for five team sport tournament is 26 and for six team sport tournament is 38. Both tournaments are considered MTTP.

1 Introduction

Currently many sports have organized the tournaments all over the world. Some sports have their own league such as soccer, american football, basket-

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ball, etc. However, each sport league may have different type of tournament. In a sport tournament, each playing team has to find sponsors to support many costs such as advertisement, traveling, contracts of players, and accommodation. The traveling costs depend on number of traveling, distance of traveling, and transportation. In this research, the objective is to study about sport tournament with minimum total number of traveling. The traveling of each team is a class of traveling tournament problem. Each team has to play against each other teams twice: home game and away game. It can be represented by some notions in graph theory as shown in section 2. Moreover, some important theorems were used in the next section as shown below.

Definition 1.1. [4] Traveling Tournament Problem (TTP) is a considered a double round robin (DRR). A scheduling to a double round robin (DRR) tournament, played by n teams, where n is a even number, consisting in a schedule where each team plays with each other twice, one game in its home and other in your opponent's home.

Definition 1.2. [7] Half Traveling tournament Problem (HTTP) is a generalization of TTP. The main defference is the concept of half double round robin (HDRR). A HDRR is a tournament where each team plays every other once in the first $n-1$ rounds.

Definition 1.3. [6] Mirrored Traveling Tournament Problem (MTTP) proposed by Ribeiro and Urrutia, is a generalization of TTP that represents the common structure in Latin America tournaments (e.g. Brazilian Soccer Championship). The main defference is the concept of mirrored double round robin (MDRR). A MDRR is a tournament where each team plays every other once in the $n-1$, followed by the same games with reversed venues in the last $n-1$ rounds. It is also a type of HTTP.

2 Problem Description

The number of traveling of each team is the total number of traveling starting from its home city and return there after the end of tournament. For the case of five team tournament, a free team (dummy team) is added. When a team has game against free team, it means no game on that week and it is called a free week. Each team has exactly two free weeks in the tournament. Any team stay at its home city during its free week. In addition, each team would not go back to its home city after away game if it did not have a home game in the week afterward.

3 Notations and Observations

3.1 Notations The five team tournament can be represented by a complete graph in five vertices. Five teams can be represented by letters A, B, C, D and

E. Each line represents two games between two corresponding teams. The number of teams is an odd number so a free team represented by letter F is added.
Then, we describe the TTP by constructing a new graph G' for each team.
The graph G'_A represents all possible traveling of team A as follow,

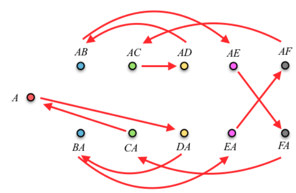

Figure 1: One possible traveling for team A

For each team X, the games between X and Y will be represented by the two vertices: XY and YX. The vertex XY represents the game between X and Y at home X. The arrow $A \to DA$ of graph G'_A represents the game between A and D at home D on the first week. The arrow $A \to DA \to BA$ represents the second week, team A has a game with team B at home B. For example, one possible traveling sequence is

$$A \to DA \to BA \to EA \to AF \to AC \to AD \to AB \to AE \to FA \to CA \to A. \quad (1)$$

After we obtain the traveling sequence of team A, we can find the traveling of team B and are given some edges from traveling sequence(1). There are only 8 empty spots left $(\ldots)$. $B \to \ldots \to BA \to \ldots \to \ldots \to \ldots \to \ldots \to AB \to \ldots \to \ldots \to B$ is a path which is gotten by traveling sequence of team A.

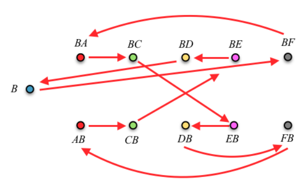

Figure 2: One possible traveling for team B

Next, the traveling sequence of team B could be

$$B \to BF \to BA \to BC \to EB \to DB \to FB \to AB \to CB \to BE \to BD \to B. \quad (2)$$

After we get the traveling sequences of team A and B, we can find the traveling of team C and are given some edges from traveling sequences(1)and (2). There are only 6 empty spots left. $C \to \ldots \to \ldots \to BC \to \ldots \to AC \to \ldots \to \ldots \to$

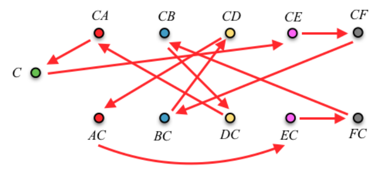

Figure 3: One possible traveling for team C

$CB \rightarrow \ldots \rightarrow CA \rightarrow C$ is a path which is gotten by traveling sequences of team A and B.

Next, the traveling sequence of team C could be

$$C \rightarrow CE \rightarrow CF \rightarrow BC \rightarrow CD \rightarrow AC \rightarrow EC \rightarrow FC \rightarrow CB \rightarrow DC \rightarrow CA \rightarrow C. \quad (3)$$

After we get the traveling sequences (1), (2)and (3), we can find the traveling of team D by filling in the only 6 empty spots left. It is a sequence

$$D \rightarrow DA \rightarrow ED \rightarrow DF \rightarrow CD \rightarrow DB \rightarrow AD \rightarrow DE \rightarrow FD \rightarrow DC \rightarrow BD \rightarrow D. \quad (4)$$

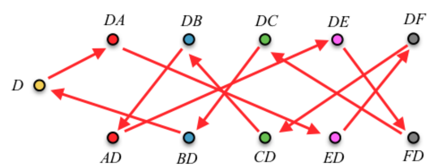

Figure 4: One possible traveling for team D

After we get the traveling sequences (1), (2), (3) and (4). The traveling sequence of team E is fixed even though there are 4 empty spots left. It is a sequence

$$E \rightarrow CE \rightarrow ED \rightarrow EA \rightarrow EB \rightarrow EF \rightarrow EC \rightarrow DE \rightarrow AE \rightarrow BE \rightarrow FE \rightarrow E. \quad (5)$$

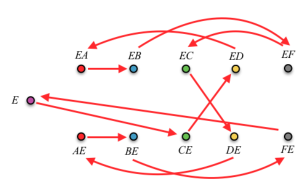

Figure 5: One possible traveling for team E

Table 1: An Example of a Scheduling

WEEK	TOURNAMENT			TRAVELING TO				
#	GAME I	GAME II	FREE TEAM	TEAM A	TEAM B	TEAM C	TEAM D	TEAM E
1	(D, A)	(C, E)	B	D	B	C	D	C
2	(B, A)	(E, D)	C	B	B	C	E	E
3	(E, A)	(B, C)	D	E	B	B	D	E
4	(E, B)	(C, D)	A	A	E	C	C	E
5	(A, C)	(D, B)	E	A	D	A	D	E
6	(A, D)	(E, C)	B	A	B	E	A	E
7	(A, B)	(D, E)	C	A	A	C	D	D
8	(A, E)	(C, B)	D	A	C	C	D	A
9	(B, E)	(D, C)	A	A	B	D	D	B
10	(C, A)	(B, D)	E	C	B	C	B	E
11				A	B	C	D	E
	Number of traveling			6	6	7	8	6

From all traveling sequences, one can make a scheduling of all five teams in eleven weeks (last week for traveling back home) as shown in Table 1.
Note (X, Y) means a game between X and Y at home X. This is also an MTTP. Then, the total number of traveling of all teams is 33.

4 Known Results

Theorem 4.1. *[7] There are* 384 *possibilities of scheduling for four team Mirrored Traveling Tournament Problem with additional condition: a game between T_i and T_j at T_j's home cannot be followed by the game between T_i and T_j at T_i's home.*

Theorem 4.2. *[7] For four team tournament, the minimum number of traveling of each team is* 4.

Lemma 4.3. *[7] The minimum number of traveling occurs when a team has all away games consecutively.*

Theorem 4.4. *[7] For four team tournament, not all team could attain a minimum number of traveling.*

Lemma 4.5. *[7] For four team tournament, Mirrored Traveling Tournament Problem cannot be made with each team traveling 5 times.*

5 Main Results

5.1 Five Team Tournament

Theorem 5.1. *For five team tournament, the minimum number of traveling of each team is* 5*.*

Proof From Lemma 4.3, a team has four away games and all away games consecutively. So, there are four traveling for four away games and plus one for going back home. Therefore, the minimum of traveling is 5. □

Theorem 5.2. *For five team tournament, not all team could attain a minimum number of traveling.*

Proof There are seven possibilities of minimum number of traveling as shown in Table 2. The first week of all choices has only one away game. If we pick any five choices for five teams from these seven possibilities, we cannot schedule the tournament because there must be 3 home teams and 2 away teams each week.

Table 2: All possible minimum traveling for each team

WEEK # / CASE #	1	2	3	4	5	6	7	8	9	10
1	Away	Away	Away	Away	Home	Home	Home	Home	Home	Home
2	Home	Away	Away	Away	Away	Home	Home	Home	Home	Home
3	Home	Home	Away	Away	Away	Away	Home	Home	Home	Home
4	Home	Home	Home	Away	Away	Away	Away	Home	Home	Home
5	Home	Home	Home	Home	Away	Away	Away	Away	Home	Home
6	Home	Home	Home	Home	Home	Away	Away	Away	Away	Home
7	Home	Home	Home	Home	Home	Home	Away	Away	Away	Away

□

However, there exists a possible scheduling for five teams which attains a minimum total number of traveling. One example is shown in Table 3.

Table 3: Schedule of five team tournament with minimum number of traveling

WEEK	TOURNAMENT			TRAVELING TO				
#	GAME I	GAME II	FREE TEAM	TEAM A	TEAM B	TEAM C	TEAM D	TEAM E
1	(D, C)	(E, B)	A	A	E	D	D	E
2	(A, B)	(E, C)	D	A	A	E	D	E
3	(A, C)	(E, D)	B	A	B	A	E	E
4	(A, D)	(B, C)	E	A	B	B	A	E
5	(A, E)	(B, D)	C	A	B	C	B	A
6	(C, D)	(B, E)	A	A	B	C	C	B
7	(B, A)	(C, E)	D	B	B	C	D	C
8	(C, A)	(D, E)	B	C	B	C	D	D
9	(D, A)	(C, B)	E	D	C	C	D	E
10	(E, A)	(D, B)	C	E	D	C	D	E
11				A	B	C	D	E
	Number of traveling			5	6	5	5	5

Then, the total number of traveling of all team is 26. It is a Mirrored Traveling Tournament Problem (MTTP).

Proposition 5.3. *For five team tournament, Mirrored Traveling Tournament Problem can be made with each team traveling 6 times.*

Proof Each team has four away games and traveling 6 times. So, not all away games are consecutive. Then we separate all away games into two parts. They are 2&2 away games, 1&3 away games and 3&1 away games.
Let A be a set of all mirrored traveling which each team travels 6 times. Then $|A| = |A_{(2,2)}| + |A_{(1,3)}| + |A_{(3,1)}|$, where $A_{(i,j)}$ is a set of i away games for the first part and j away games for the second part.
Hence, $|A_{(2,2)}| = 9$, $|A_{(1,3)}| = 9$ and $|A_{(3,1)}| = 9$.
Thus, $|A| = |A_{(2,2)}| + |A_{(1,3)}| + |A_{(3,1)}| = 27$.
It means that we have $\binom{27}{5}$ possibilities, but not all cases make a possible schedule. $\square$

However, there exists a possible scheduling for five teams such that each team travels exactly 6 times as shown in Table 4.

Table 4: Schedule of five team tournament with each team traveling 6 times

WEEK #	TOURNAMENT GAME I	GAME II	FREE TEAM	TRAVELING TO TEAM A	TEAM B	TEAM C	TEAM D	TEAM E
1	(C,D)	(E,B)	A	A	E	C	C	E
2	(A,B)	(E,D)	C	A	A	C	E	E
3	(A,E)	(C,B)	D	A	C	C	D	A
4	(D,A)	(C,E)	B	D	B	C	D	C
5	(C,A)	(D,B)	E	C	D	C	D	E
6	(D,C)	(B,E)	A	A	B	D	D	B
7	(B,A)	(D,E)	C	B	B	C	D	D
8	(E,A)	(B,C)	D	E	B	B	D	E
9	(A,D)	(E,C)	B	A	B	E	A	E
10	(A,C)	(B,D)	E	A	B	A	B	E
11				A	B	C	D	E
	Number of traveling			6	6	6	6	6

Since each team travels exactly 6 times, the total number of traveling of all teams is 30. It is also a Mirrored Traveling Tournament Problem (MTTP).

5.2 Six Team Tournament

We now consider a case of six team sport tournament. There are the following results. In this part, we would like to show only results because most theorems are similar to theorems of five team tournament. For six team tournament, not all teams could attain a minimum number of traveling. However, there exists a possible scheduling for six teams which attain a minimum total number of traveling.

One example is shown in Table 5 and 6. The total number of traveling of all team is 38. It is also a Mirrored Traveling Tournament Problem(MTTP).

Proposition 5.4. *For six team tournament, Mirrored Traveling Tournament Problem cannot be made with each team traveling 7 times.*

Proof Each team has five away games and traveling 7 times. So, not all away games are consecutive. Then we separate all away games into two parts. They are 1&4 away games, 2&3 away games, 3&2 away games and 4&1 away games. Let A be a set of all mirrored traveling which each team travels 7 times. Then $|A| = |A_{(1,4)}| + |A_{(2,3)}| + |A_{(3,2)}| + |A_{(4,1)}|$, where $A_{(i,j)}$ is a set of i away games for the first part and j away games for the second part.
Since we consider only MTTP, each set must have all five consecutive home games in between two parts of away games. Hence, $|A_{(1,4)}| = 1$, $|A_{(2,3)}| = 1$, $|A_{(3,2)}| = 1$ and $|A_{(4,1)}| = 1$. Thus, $|A| = |A_{(1,4)}| + |A_{(2,3)}| + |A_{(3,2)}| +$

Table 5: Schedule of six team tournament with minimum number of traveling

week	tournament		
#	game i	game ii	game iii
1	(B, A)	(E, C)	(F, D)
2	(E, A)	(F, C)	(B, D)
3	(F, A)	(D, C)	(E, B)
4	(C, A)	(F, B)	(D, E)
5	(D, A)	(C, B)	(F, E)
6	(A, B)	(C, E)	(D, F)
7	(A, E)	(C, F)	(D, B)
8	(A, F)	(D, C)	(B, E)
9	(A, C)	(B, F)	(E, D)
10	(A, D)	(C, B)	(E, F)

$|A_{(4,1)}| = 4$. It means that we have only 4 possibilities, but there are six teams. Therefore, the tournament for six teams cannot be made with each team traveling 7 times. □

Table 6: Traveling of six team tournament with minimum number of traveling

week	traveling to					
#	team A	team B	team C	team D	team E	team F
1	B	B	E	F	E	F
2	E	B	F	B	E	F
3	F	E	D	D	E	F
4	C	F	C	D	D	F
5	D	C	C	D	F	F
6	A	A	C	D	C	D
7	A	D	C	D	A	C
8	A	B	C	C	B	A
9	A	B	A	E	E	B
10	A	B	B	A	E	E
11	A	B	C	D	E	F
number of traveling	6	6	7	7	6	6

6 Conclusion

We can always schedule the sport tournaments with minimum total number of traveling for five and six teams. The minimum total number of all team traveling for five team sport tournament is 26 and for six team sport tournament is 38. Both tournaments are considered MTTP. Moreover, scheduling for five team MTTP can be made fair to every team by setting equal minimum number of traveling of each team.

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