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# THE LOCATING-CHROMATIC NUMBER FOR CORONA PRODUCT OF GRAPHS

**Edy T. Baskoro**[*] and **Ira A. Purwasih**[†]

*Combinatorial Mathematics Research Group*
*Faculty of Mathematics and Natural Sciences*
*Institut Teknologi Bandung (ITB)*
*Jalan Ganesa 10 Bandung, Indonesia*
*e-mail:* [*] *ebaskoro@math.itb.ac.id,* [†] *ira.apni.p@students.itb.ac.id*

**Abstract**

The locating-chromatic number of a graph $G$ can be defined as the cardinality of a minimum ordered partition $\Pi$ of the vertex set $V(G)$ such that every vertex in $G$ has the different coordinates with respect to $\Pi$ and every two adjacent vertices in $G$ are not included in the same partition class. In this case, the coordinate of a vertex $v$ is defined as the distances from vertex $v$ to the ordered partition classes in $\Pi$. In this paper, we discuss the locating-chromatic number for a corona product of two graphs.

## 1 Introduction

The concept of graph locating-chromatic number was introduced by Chartrand, Erwin, Henning, Slater and Zhang [3] in 2002, as a marriage between two previous big concepts in graph, namely graph coloring and graph partition dimension. Let $G = (V, E)$ be a connected graph. The *distance* $d(u, v)$ between vertices $u$ and $v$ in $G$ is the length of a shortest path connecting $u$ and $v$ in $G$. For $v \in V(G)$ and $S \subset E(G)$, the *distance* $d(v, S)$ from $u$ to $S$ is defined as $\min\{d(v, x)|x \in S\}$. In particular, if $d(x, S) \neq d(y, S)$ then we shall say that $x$ and $y$ are *distinguished* by $S$ or $x$ and $y$ are *distinguishable*. Let $c$ be a *proper $k$-coloring* of $V(G)$ which induces an ordered partition $\Pi = \{S_1, S_2, \cdots, S_k\}$ of $V(G)$, where $S_i$ is the set of all vertices colored by $i$ in $G$. The *color code* $c_\Pi(v)$ of vertex $v$ is the ordered $k$-tuple $(d(v, S_1), d(v, S_2), \cdots, d(v, S_k))$, where $d(v, S_i) = \min\{d(v, x)|x \in S_i\}$ for $1 \leq i \leq k$. If every two vertices have different

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color codes then $c$ is called a *locating $k$-coloring* of $G$. The *locating-chromatic number* of graph $G$, denoted by $\chi_L(G)$, is the smallest integer $k$ such that $G$ has a locating $k$-coloring.

The problem of determining the locating-chromatic number of any general graph is an NP-hard problem. This means that no efficient algorithm to determine the locating-chromatic number of any given graph. Therefore, some heuristics methods have been developed to determine these numbers. Some studies have also been done by applying to certain classes of graphs, such as paths, cycles, certain trees and others. Characterization studies for all graphs having a specific locating-chromatic number have also carried out. However, the results are still very limited and not yet satisfactory. Some are presented below.

Chartrand *et al.* [3] have determined the locating-chromatic numbers of some well-known classes such as paths, cycles, complete multipartite graphs and double stars. Furthermore, in [4] they showed the existence of a tree of order $n(\geq 5)$ having the locating-chromatic number $k$ if and only if $k \in \{3, 4, ..., n-2, n\}$.

In [2], Asmiati *et al.* have managed to determine the locating-chromatic number for a special class of tree, namely an amalgamation of $n$ stars that are not necessarily isomorphic. Furthermore, in [1], they determined the locating-chromatic number for a firecrackers graph, i.e. a special tree constructed from $n$ stars by connecting one leaf from each star to form a path $P_n$.

For any given graphs $G$ and $H$, define the *corona product* $G \odot H$ between $G$ and $H$ as the graph obtained from $G$ and $H$ by taking one copy of $G$ and $|V(G)|$ copies of $H$ and then joining all the vertices of the $i^{th}$-copy of $H$ with the $i^{th}$-vertex of $G$. Therefore, if $V(G) = \{x_1, x_2, \cdots, x_n\}$ and $V(H) = \{a_1, a_2, \cdots, a_m\}$ then $V(G \odot H) = V(G) \cup V(H_1) \cup V(H_2) \cup \cdots \cup V(H_n)$, where $V(H_i) = \{a_{ij} | 1 \leq j \leq m\}$ is the vertex-set of the $i^{th}$-copy of $H$, and $E(G \odot H) = E(G) \cup \{x_i a_{ij} | 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{a_{ij} a_{ik} | 1 \leq i \leq n, 1 \leq j < k \leq m,$ whenever $a_j a_k \in E(H)\}$. For simplicity, for each $i$, let $A_i = \{x_i\} \cup \{a_{ij} | 1 \leq j \leq m\}$. In this paper we determine the locating-chromatic number of $G \odot H$.

The following lemma is useful in determining the locating-chromatic number of a graph $G$. This lemma is a modification of a similar lemma derived by Chartrand *et al.* (2003) in [5].

**Lemma 1.** *Let $G$ be a connected non trivial graph. Let $c$ be a locating coloring for $G$ and $u, v \in V(G)$. If $d(u, w) = d(v, w)$ for all $w \in V(G) - \{u, v\}$, then $u$ and $v$ must be in different color.*

## 2 Results

In this section, we will present the upper bound of the locating-chromatic number for corona product of two connected graph $G \odot H$ and the exact value of the locating-chromatic number for corona product of certain graphs.

Let $G$ and $H$ be connected graphs. Let $H_i$ be the $i^{th}$-copy of $H$ in $G \odot H$. Because any two vertices $u$ and $v$ of $H_i$ have same distance from other vertices, then by Lemma 1, we have the following lemma.

**Lemma 2.** *Let $G$ and $H$ be connected graphs. Let $H_i$ be the $i^{th}$-copy of $H$ in $G \odot H$. Then, any two vertices $u$ and $v$ of $H_i$ can only be distinguished by a set $R$ such that $R \cap V(H_i) \neq \emptyset$.*

Firstly we give the upper bound of the locating-chromatic number for corona product of two connected graphs $G \odot H$.

**Theorem 1.** *Let $G$ and $H$ be two connected graphs with diameter of $H \leq 2$ then $\chi_L(G \odot H) \leq \chi_L(G) + \chi_L(H)$.*

**Proof**. Let $\Pi_G$ and $\Pi_H$ be minimum locating colorings of $G$ and $H$, respectively. Let $|V(G)| = n$. For $i = 1, 2, \cdots, n$, color/partition all the vertices of each $H_i$ according to $\Pi_H$, say $\{V(H_i)^1, V(H_i)^2, \cdots, V(H_i)^s\}$, where $s = pd(H)$. Now, consider the coloring/partition $\Pi = \Pi_1 \cup \Pi_2$ on $G \odot H$, where $\Pi_1 = \{\cup_{i=1}^n V(H_i)^1, \cup_{i=1}^n V(H_i)^2, \cdots, \cup_{i=1}^n V(H_i)^s\}$ and $\Pi_2 = \Pi_G$. Next, we will show that $\Pi$ is a locating coloring of $G \odot H$. Note that since the diameter of $H$ is at most 2, then the distance of any two vertices $u, v \in V(H_i)$, for any $i$, under the corona graph $G \odot H$ is the same as its distance under the original graph $H$. Therefore, if the vertices $u, v \in V(H_i)$, for any $i$, are distinguishable by $\Pi_H$ then they are distinguishable too by $\Pi_1$. Let $u$ and $v$ be any two vertices of $G \odot H$. If $u, v \in V(H_i)$ then they will be clearly distinguished by $\cup_{i=1}^n V(H_i)^t$ for some $t$. If $u, v \in V(G)$ then they will be distinguished by some set in $\Pi_G$. Now, assume that $u \in V(H_i)$ and $v \in V(G)$. If $u \in \cup_{i=1}^n V(H_i)^t$ for some $t$, then the distances between $u$ and $v$ to $\cup_{i=1}^n V(H_i)^t$ is 0 and 1, respectively. Therefore, $u$ and $v$ are distinguished. Now, the only case we have not considered is $u \in V(H_i)$ and $v \in V(H_j)$, for $i \neq j$. If $u, v \in \cup_{i=1}^n V(H_i)^t$ for some $t$ then $u, v$ are distinguished by some set in $\Pi_G$ since $\Pi_G$ is a locating coloring for $G$. □

Now, we consider the corona product $G \cong P_n \odot K_m$, where $P_n$ represents a path of order $n$ and $K_m$ is the complete graph on $m$ vertices. Let $V(G) = \{x_1, x_2, \cdots, x_n\} \cup V(H_1) \cup V(H_2) \cup \cdots \cup V(H_n)$, where $V(H_i) = \{a_{ij} | 1 \leq j \leq m\}$ is the vertex-set of the $i^{th}$-copy of $K_m$. We will show that the upper bound of Theorem 1 is satisfied by $\chi_L(P_n \odot K_m)$ provided $n \geq 2(m+2)+1$.

**Theorem 2.** *For $n \geq 1, m \geq 2$, the locating-chromatic number of $P_n \odot K_m$ is as follows:*

$$\chi_L(P_n \odot K_m) = \begin{cases} m+1, & \text{if } n = 1, \\ m+2, & \text{if } 2 \le n \le 2(m+2), \\ m+3, & \text{if } n \ge 2(m+2)+1. \end{cases}$$

**Proof**. By Lemma 1, every two vertices in $V(H_i)$ must be in different color classes. Since $x_i$ is adjacent to all vertices of $H_i$, then $x_i$ must be in a different color class other than all the color classes in $V(H_i)$. Therefore, $\chi_L(P_n \odot K_m) \ge m+1$. If $n = 1$ then it is clear that $\chi_L(P_n \odot K_m) = m+1$. If $n \ge 2$ then in order to have different color codes for each $x_i$ we must have $\chi_L(P_n \odot K_m) \ge m+2$. Now, consider a locating $(m+2)$-coloring $c$ on $G \cong P_n \odot K_m$, for $n, m \ge 2$. For each $i$, let $A_i = \{x_i\} \cup V(H_i)$. Then, we have the following facts.

**Fact 1**. No two integers $i$ and $j$ such that $c(A_i) = c(A_j)$ but $c(x_i) \ne c(x_j)$. This is true since if $j = i+1$ and $i = 1$ then $x_i$ and $a_{jk}$, for some $k$, will have the same color codes. If $j = i+1$ and $i \ne 1$ then either the vertices ($x_j$ and $a_{ik}$) or ($a_{ik}$ and $a_{jl}$), will have the same color codes, for some $k, l$. Now, let $j \ge i+2$ and w.l.o.g. let $c(A_i) = [1, m+1]$. Then, to distinguish vertex $x_i$ with the vertices of $H_j$ we have that $x_{i-1} \in S_m$ or $x_{i+1} \in S_m$. Similarly, we have that $x_{j-1} \in S_m$ or $x_{j+1} \in S_m$. Therefore, there exists two vertices $a_{ik}$ and $a_{jl}$ with the same color codes, for some $k, l$.

**Fact 2**. No three integers $i, j$ and $k$ such that $c(A_i) = c(A_j) = c(A_k)$ and $c(x_i) = c(x_j) = c(x_k)$. Without loss of generality, let $c(A_i) = [1, m+1]$ and $c(x_i) = m$. Then, if $i < j < k$ then $j \ge i+2$ and $k \ge j+2$. Therefore, $d(x_i, S_m) = 1$ or 2. Similarly for $x_j$ and $x_k$. This implies that the color codes of two vertices of $\{x_i, x_j, x_k\}$ will be the same, a contradiction. Therefore, this fact holds.

From these two facts, we conclude that in any locating $(m+2)$-coloring $c$ on $G$, if $c(A_i) = c(A_j)$, for $i < j$, then $c(x_i) = c(x_j)$; and we cannot have three $A_i$s with the same $c(A_i)$. Therefore, a locating $(m+2)$-coloring $c$ can only exists on $G$ if $n \le 2(m+2)$. To show the coloring, let us define the mapping $c : V(P_n \odot K_m) \to [1, m+2]$ such that:

$$c(x_i) = \begin{cases} i, & \text{if } 1 \le i \le m+1; \\ 1, & \text{if } i = m+2; \\ m+2, & \text{if } i = m+3 \text{ or } i = m + \lceil m/2 \rceil + 4; \\ 3 + 2(i-m-4), & \text{if } m \text{ is even, } m+4 \le i \le m + \lceil m/2 \rceil + 3; \\ 2 + 2(i-m-\lceil m/2 \rceil - 5), & \text{if } m \text{ is even, } m + \lceil m/2 \rceil + 5 \le i \le n; \\ 2 + 2(i-m-4), & \text{if } m \text{ is odd, } m+4 \le i \le m + \lceil m/2 \rceil + 3; \\ 3 + 2(i-m-\lceil m/2 \rceil - 5), & \text{if } m \text{ is odd, } m + \lceil m/2 \rceil + 5 \le i \le n. \end{cases}$$

$$c(V(H_i)) = \begin{cases} [1, m+2] - \{1, m+2\}, & \text{if } i = m+3 \text{ or } i = m + \lceil m/2 \rceil + 4; \\ [1, m+2] - \{c(x_i), c(x_i)+1\}, & \text{otherwise.} \end{cases}$$

We will show that $c$ is a locating coloring on $G \cong P_n \odot K_m$, if $2 \leq n \leq 2(m+2)$. Let $u, v$ be two vertices in the same color class. If $u = x_i$ and $v = x_j$ for some $i, j$ then they will have different color codes. If $u = x_i$ and $v = a_{jk}$ for some $i, j, k$ then $1 = d(u, S_t) < d(v, S_t)$ where $t = c(j) + 1 \pmod{m+2}$. If $u = a_{ik}$ and $v = a_{jl}$ for some $i, j, k, l$ then $d(u, S_t) \neq d(v, S_t)$ where $t = c(j) + 1 \pmod{m+2}$. Therefore $c$ is a locating coloring on $G$.

Now, consider the case of $n \geq 2(m+2)+1$. To show the upper bound for this case, define the mapping $c : V(P_n \odot K_m) \to [1, m+3]$ such that:

$$c(x_i) = \begin{cases} m+3 & \text{if } i = 1, \\ 2 & \text{if } i \text{ is even}, \\ 3 & \text{if } i \text{ is odd and } i \neq 1, \end{cases}$$

$$c(V(H_i)) = \begin{cases} [1, m] & \text{if } i = 1, \\ [1, m+2] - \{2, 3\} & \text{otherwise.} \end{cases}$$

In order to show $c$ is a locating coloring on $G$, we need only to consider the case of vertices $u, v$ such that $d(u, x_1) = d(v, x_1)$. This implies that $u = a_{ik}$ and $v = x_{i+1}$ for some $i, k$. If $i > 1$ then $u$ and $v$ must be in different color classes under $c$. If $i = 1$ then $d(u, S_{m+1}) \neq d(v, S_{m+1})$. Therefore $c$ is a locating coloring on $G$. □

Now, we consider the corona product $G \cong P_n \odot \overline{K}_m$, where $P_n$ represents a path of order $n$ and $\overline{K}_m$ is the complement of the complete graph on $m$ vertices. Let the vertex-set $V(G) = \{x_i | 1 \leq i \leq n\} \cup \{a_{ij} | 1 \leq i \leq n, 1 \leq j \leq m\}$ and the edge-set $E(G) = \{x_{i-1}x_i | 2 \leq i \leq n\} \cup \{x_i a_{ij} | 1 \leq i \leq n, 1 \leq j \leq m\}$.

**Theorem 3.** *For $n, m \geq 1$, the locating-chromatic number of $P_n \odot \overline{K}_m$ is as follows:*

$$\chi_L(P_n \odot \overline{K}_1) = \begin{cases} 2, & \text{if } n = 1 \\ 3, & \text{if } 2 \leq n \leq 6 \\ 4, & \text{if } n \geq 7, \end{cases}$$

$$\chi_L(P_n \odot \overline{K}_m) = \begin{cases} m+1, & \text{if } m \geq 2 \text{ and } 1 \leq n \leq m+1, \\ m+2, & \text{if } m \geq 2 \text{ and } n \geq m+2. \end{cases}$$

**Proof**. Let $V(P_n \odot \overline{K}_m) = \{x_1, x_2, \cdots, x_n\} \cup V(H_1) \cup V(H_2) \cup \cdots \cup V(H_n)$, where $V(H_i) = \{a_{ij} | 1 \leq j \leq m\}$ is the vertex-set of the $i^{th}$-copy of $\overline{K}_m$. Now,

consider the following two cases.

**Case 1**. $m = 1$.
For $n = 1$, we have $P_1 \odot \overline{K}_1 \cong P_2$, then it is clear that $\chi_L(P_1 \odot \overline{K}_1) = 2$. Now, let $m = 1$ and $2 \leq n \leq 6$, then clearly $\chi_L(P_n \odot \overline{K}_1) \geq 3$. Figure 1 shows the locating 3-coloring for this case, therefore $\chi_L(P_n \odot \overline{K}_1) = 3$, for $2 \leq n \leq 6$.

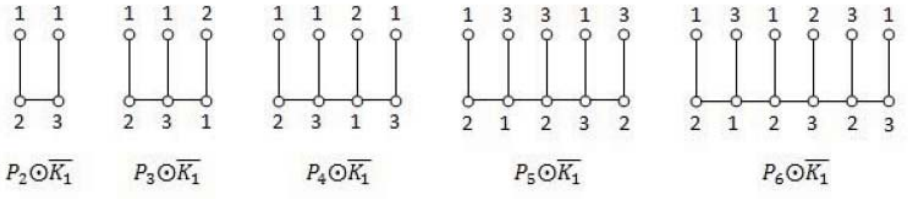



Figure 1: The locating 3-coloring of $P_n \odot \overline{K}_1$ for $2 \leq n \leq 6$.

If $n \geq 7$ then define $c : V(P_n \odot \overline{K}_1) \to [1, 4]$ such that:

$$c(x_i) = \begin{cases} 4 & \text{if } i = 1, \\ 3 & \text{if } i \text{ is odd and } i \neq 1, \\ 2 & \text{if } i \text{ is even}, \end{cases}$$

$$c(a_{i1}) = \begin{cases} 1 & \text{if } i = 1 \text{ or } i \text{ is even}, \\ 2 & \text{otherwise.} \end{cases}$$

The mapping $c$ is a locating coloring on $G \cong P_n \odot \overline{K}_1$, since if $d(u, x_1) = d(v, x_1)$ then $u = a_{i1}$ and $v = x_{i+1}$ for some $i$. This implies that $u, v \in S_2$ and $1 = d(v, S_1) < d(u, S_1) = 2$. Therefore the color codes of $u$ and $v$ are different. To show that $\chi_L(G) \geq 4$ for $n \geq 7$, assume for a contradiction $\chi_L(G) = 3$, for $n \geq 7$. Let us call a vertex $x$ with distance 1 to other two color classes by a *dominant vertex* in $G$. Then, there must be three dominant vertices in $G$, otherwise there will be two vertices with the same color code. These three dominant vertices must be in different color classes and they must be the vertices $x_i, x_j, x_k$, for some $i, j, k$ and $1 \leq i < j < k \leq n$. Furthermore, $d(x_i, x_j)$ and $d(x_j, x_k)$ must be odd; and both are at most 3, since w.l.o.g. if $d(x_i, x_j) \geq 5$ then the color codes of $x_{i+2}$ and $a_{(i+1)1}$ are the same, a contradiction. Next, one of these distances must be 1. Since otherwise (w.l.o.g. if $d(x_i, x_j) = d(x_j, x_k) = 3$), then either the color codes of $x_{j+1}$ and $a_{j1}$ are the same or the color codes of $x_{j-1}$ and $a_{j1}$ are the same, a contradiction. Therefore, $n < 7$, a contradiction. This concludes the proof of this case. Note that this case has been also proved in [1].

**Case 2**. $m \geq 2$.
Let $1 \leq n \leq m+1$. Since every two vertices $a_{ij}$ and $a_{ik}$, for $1 \leq i \leq n$ and $1 \leq j < k \leq m$, we have $d(a_{ij}, x) = d(a_{ik}, x)$ for every $x \in P_n \odot \overline{K}_m - \{a_{ij}, a_{ik}\}$, then by Lemma 1, $\chi_L(P_n \odot \overline{K}_m) \geq m+1$. For the upper bound, we construct a mapping

$$c : V(P_n \odot \overline{K}_m) \to [1, m+1] \text{ such that}$$

$$c(x_i) = i, c(V(H_i)) = [1, m+1] - \{i\}, \text{for each } i.$$

Then, it is easy to see that $c$ is a locating-coloring of $P_n \odot \overline{K}_m$, for $m \geq 2$ and $1 \leq n \leq m+1$.

Now, let $n \geq m+2$. Again, by Lemma 1, $\chi_L(P_n \odot \overline{K}_m) \geq m+1$. For a contradiction, suppose $\chi_L(P_n \odot \overline{K}_m) = m+1$. Since $n \geq m+2$, then there are two vertices $x_i, x_j \in P_n \odot \overline{K}_m$ have the same color. Since each of these two vertices adjacent to $m$ different color vertices, then the color codes of these two vertices are the same, a contradiction. Hence, $\chi_L(P_n \odot \overline{K}_m) \geq m+2$. For the upper bound, we construct the following mapping $c : V(P_n \odot \overline{K}_m) \to [1, m+2]$ :

$$c(x_i) = \begin{cases} m+2 & \text{if } i = 1, \\ m+1 & \text{if } i \text{ is odd and } i \neq 1, \\ m & \text{if } i \text{ is even,} \end{cases}$$

$$c(a_{ij}) = \begin{cases} j+1 & \text{if } i \text{ is even and } j = m, \\ j & \text{otherwise.} \end{cases}$$

To show that $c$ is a locating coloring of $G \cong P_n \odot \overline{K}_m$, we only need to consider any two distinct vertices $u$ and $v$ in $G$ satisfying $d(u, x_1) = d(v, x_1)$. Then, it implies that $u = a_{ij}$ and $v = x_{i+1}$ for some $i$ and $j$. In this case, we have $1 = d(v, S_{m-1}) < d(u, S_{m-1}) = 2$. Therefore, the color codes of $u$ and $v$ are different. Therefore $c$ is a locating coloring on $G$. □

Now, we consider the corona product $C_n \odot \overline{K}_m$, where $C_n$ represents a cycle of order $n$ and $\overline{K}_m$ is the complement of the complete graph on $m$ vertices.

**Theorem 4.** *For $n, m \geq 1$, the locating-chromatic number of $C_n \odot \overline{K}_m$ is as follows:*

$$\chi_L(C_n \odot \overline{K}_1) = \begin{cases} 3, & \textit{if } 3 \leq n \leq 5 \\ 4, & \textit{if } n \geq 6, \end{cases}$$

$$\chi_L(C_n \odot \overline{K}_m) = \begin{cases} m+1, & \textit{if } m \geq 2 \textit{ and } 3 \leq n \leq m+1, \\ m+2, & \textit{if } m \geq 2 \textit{ and } n \geq m+2. \end{cases}$$

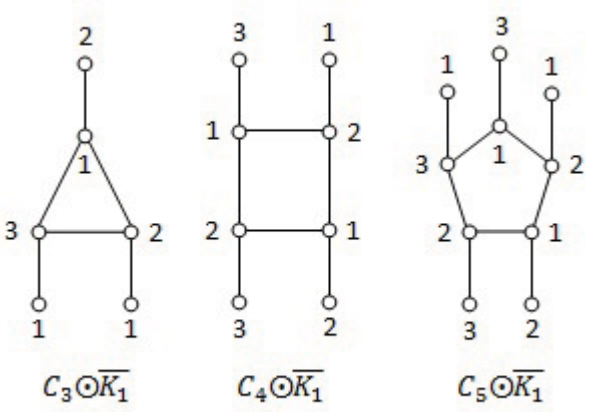



Figure 2: The locating 3-coloring of $C_n \odot \overline{K}_1$ for $3 \leq n \leq 5$.

**Proof**. Let $V(C_n \odot \overline{K}_m) = \{x_1, x_2, \cdots, x_n\} \cup V(H_1) \cup V(H_2) \cup \cdots \cup V(H_n)$, where $V(H_i) = \{a_{ij} | 1 \leq j \leq m\}$ is the vertex-set of the $i^{th}$-copy of $\overline{K}_m$. By Lemma 1, every two vertices in $V(H_i)$ must be in different color classes. Since $x_i$ is adjacent to all vertices of $H_i$, then $x_i$ must be in a different color class other than all the color classes in $V(H_i)$. Therefore, $\chi_L(C_n \odot \overline{K}_m) \geq m + 1$. Now, consider the following two cases.

**Case 1**. $m = 1$.
For $3 \leq n \leq 5$, then it is clear that $\chi_L(C_n \odot \overline{K}_1) \geq 3$. Figure 2 shows the locating 3-coloring for this case, therefore $\chi_L(C_n \odot \overline{K}_1) = 3$, for $3 \leq n \leq 5$.
If $n \geq 6$ then define $c : V(C_n \odot \overline{K}_1) \rightarrow [1, 4]$ such that:

$$c(x_i) = \begin{cases} 4 & \text{if } i = 1, \\ 3 & \text{if } (n \text{ and } i \text{ are odd, } i \neq 1) \text{ or} \\ & (n \text{ is even, } i \text{ is odd, } 1 < i \leq 2\lfloor \frac{n}{4} \rfloor + 1), \\ 2 & \text{if } (n \text{ is odd and } i \text{ is even}) \text{ or} \\ & (n \text{ is even, } i \text{ is odd and } i > 2\lfloor \frac{n}{4} \rfloor + 1) \text{ or} \\ & (n \text{ is even, } i \text{ is even and } i \leq \frac{n}{2}), \\ 1 & \text{if } n, i \text{ are even and } i \geq \frac{n}{2} + 1 \end{cases}$$

$$c(a_{i1}) = \begin{cases} 1 & \text{if } (n \text{ is odd and all } i) \text{ or} \\ & (n \text{ is even, } i \leq 2\lfloor \frac{n}{4} \rfloor + 1) \\ 3 & \text{if } n \text{ is even and } i > 2\lfloor \frac{n}{4} \rfloor + 1. \end{cases}$$

The mapping $c$ is a locating coloring on $G \cong C_n \odot \overline{K}_1$, for $n \geq 6$ since if $d(u, x_1) = d(v, x_1)$ then (i) $u = a_{i1}$, $v = x_{i+1}$ for some $i$, (ii) $u = x_i$, $v = x_{n+1-i}$, (iii) $u = a_{i1}$, $v = x_{n+i-i}$, or (iv) $u = x_i$, $v = a_{(n+1-i)1}$. If either (i) or (ii) holds then the vertices $u$ and $v$ are in the different color classes. If (iii) holds then $1 = d(v, S_2) < d(u, S_2) = 2$ or $u$ and $v$ are in different color classes. If (iv) holds then $1 = d(u, S_2) < d(v, S_2) = 2$ or $u$ and $v$ are in different color classes. Therefore, in any case above the color codes of $u$ and $v$ are different. Therefore, $c$ is a locating coloring.

To show that $\chi_L(G) \geq 4$ for $n \geq 6$, assume for a contradiction $\chi_L(G) = 3$, for $n \geq 6$. Recall, a vertex $x$ is *dominant* if the distances of $x$ to all other color classes are 1. Then, if $n$ is odd then there are exactly three dominant vertices in $G$, but if $n$ is even then there are exactly two dominant vertices. Otherwise, there will be two vertices in $G$ with the same color code. These dominant vertices must be in different color classes and they must be the vertices $x_i, x_j, x_k$, for some $i, j, k$ and $1 \leq i < j < k \leq n$. Let $l_{ij}, l_{jk}$, and $l_{ki}$ be the length of shortest paths connecting $x_i$ to $x_j$, $x_j$ to $x_k$, or $x_k$ to $x_i$, which do not pass $x_k$, $x_i$, or $x_j$, respectively. Then, $l_{ij}$, $l_{jk}$ and $l_{ki}$ must be odd; and these are at most 3, since without loss of generality if $l_{ij} \geq 5$ then the color codes of $x_{i+2}$ and $a_{(i+1)1}$ are the same, a contradiction. If $n$ is even then the distance between these two dominant vertices is either 1 or 3. If $n$ is odd then one of $l_{ij}$, $l_{jk}$ or $l_{ki}$ is 1. Since otherwise (w.l.o.g. if $l_{ij} = l_{jk} = 3$), then either the color codes of $x_{j+1}$ and $a_{j1}$ are the same or the color codes of $x_{j-1}$ and $a_{j1}$ are the same, a contradiction. Therefore, $n \leq 7$. However, if $n = 6$ then two neighbors of the dominant vertex $x_i$ will have the same color code. If $n = 7$ and $l_{ij} = l_{jk} = 1$ then the color codes of $x_{i-2}$ and $a_{(i-1)1}$ are the same, a contradiction. If $n = 7$ and $l_{ij} = 1$ and $l_{jk} = 3$ then the color codes of two neighbors of the dominant vertex $x_k$ are the same, a contradiction. Therefore, this concludes that $\chi_L(G) \geq 4$ for $n \geq 6$.

**Case 2**. $m > 1$.
For $3 \leq n \leq m + 1$, define $c_1 : V(C_n \odot \overline{K}_m) \to [1, m + 1]$ such that: $c_1(x_i) = i$ and $c_1(V(H_i)) = [1, m + 1] - \{i\}$, for each $i$, where $c_1(V(H_i)) = \{c_1(a_{i1}), c_1(a_{i2}), \cdots, c_1(a_{im})\}$. It is clear that $c_1$ is a locating coloring of $C_n \odot \overline{K}_m$. Now, if $n \geq m+2$ then it is clear that $\chi_L(C_n \odot \overline{K}_m) \geq m+2$, since otherwise there are two distinct vertices $x_i$ and $x_j$ in the same partition class. This will imply that the color codes of these vertices are the same, a contradiction. To show the upper bound, define $c_2 : V(C_n \odot \overline{K}_m) \to [1, m+2]$ such that: $c_2(x_1) = m + 2, c_2(H_1) = [1, m]$, and $c_2(x_i) = c(x_i)$, and $c_2(V(H_i)) = [1, m + 1] - \{c_2(x_i)\}$, for all $i > 1$, where $c$ is the locating coloring in Case 1. It can be verified that $c_2$ is a locating coloring of $C_n \odot \overline{K}_m$ for $n \geq m + 2$ and $m \geq 2$. □

Now, we consider the corona product $K_n \odot \overline{K}_m$, where $K_n$ represents a complete graph of order $n$ and $\overline{K}_m$ is the complement of the complete graph on $m$ vertices.

**Theorem 5.** *For $n, m \geq 1$, the locating-chromatic number of $K_n \odot \overline{K}_m$ is as follows:*

$$\chi_L(K_n \odot \overline{K}_m) = \begin{cases} m+1, & \text{if } n \leq m+1 \\ n, & \text{if } n > m+1 \end{cases}$$

**Proof**. Let $V(K_n \odot \overline{K}_m) = \{x_1, x_2, \cdots, x_n\} \cup V(H_1) \cup V(H_2) \cup \cdots \cup V(H_n)$, where $V(H_i) = \{a_{ij} | 1 \leq j \leq m\}$ is the vertex-set of the $i^{th}$-copy of $\overline{K}_m$. By Lemma 1, every two vertices in $V(H_i)$ must be in different color classes. Since $x_i$ is adjacent to all vertices of $H_i$, then $x_i$ must be in a different color class other than all the color classes in $V(H_i)$. Therefore, $\chi_L(K_n \odot \overline{K}_m) \geq m+1$. If $n \leq m+1$ then define a mapping $c$ on $V(K_n \odot \overline{K}_m)$ such that $c(x_i) = i$ and $c(V(H_i)) = [1, m+1] - \{i\}$ for each $i$. This mapping $c$ is a locating coloring. Therefore, $\chi_L(K_n \odot \overline{K}_m) = m+1$ for $n \leq m+1$.

Now, consider $n > m+1$. Since every two $x_i$s must be in different classes then $\chi_L(K_n \odot \overline{K}_m) \geq n$. By constructing a mapping $c$ such that

$$c(x_i) = i,$$

$$c(V(H_i)) = \begin{cases} [1, m+1] - \{i\} & \text{if } i \leq m+1, \\ [1, m] & \text{otherwise,} \end{cases}$$

we will show that $c$ is a locating coloring. If $d(u, x_1) = d(v, x_1)$ then we have: (i) $u = x_k$ and $v = x_l$ for some $k, l$; (ii) $u = x_i$ and $v = a_{1k}$ for some $i \neq 1$ and $k$, or (iii) $u = a_{it}$ and $v = a_{js}$ for some $i, j, t, s$. If (i) holds then $u$ and $v$ must be in different classes. If (ii) holds then $d(u, S_{i-1}) \neq d(v, S_{i-1})$. If (iii) holds then $d(u, S_i) \neq d(v, S_i)$. Therefore, $c$ is a locating coloring and so $\chi_L(K_n \odot K_m) = n$. □

Now, we consider the corona product $K_n \odot K_m$, where $K_n$ represents a complete graph of order $n$ and $K_m$ is the complete graph on $n$ vertices.

**Theorem 6.** *For $n, m \geq 1$, the locating-chromatic number of $K_n \odot K_m$ is as follows:*

$$\chi_L(K_n \odot K_m) = \begin{cases} m+1, & \text{if } n = 1 \\ m+2, & \text{if } 2 \leq n \leq m+2 \\ n, & \text{if } n > m+2. \end{cases}$$

**Proof**. Let $V(K_n \odot K_m) = \{x_1, x_2, \cdots, x_n\} \cup V(H_1) \cup V(H_2) \cup \cdots \cup V(H_n)$, where $V(H_i) = \{a_{ij} | 1 \leq j \leq m\}$ is the vertex-set of the $i^{th}$-copy of $K_m$. By Lemma 1, every two vertices in $V(H_i)$ must be in different color classes. Furthermore, $x_i$ is adjacent to all vertices of $H_i$. Then, $x_i$ is in a different color class other than all color classes in $V(H_i)$. Therefore, $\chi_L(K_n \odot K_m) \geq m+1$. If $n = 1$ then it is clear that $\chi_L(K_n \odot K_m) = m+1$. If $2 \leq n \leq m+2$ then $\chi_L(K_n \odot K_m) \geq m+2$. Let us define a mapping $c$ satisfying:

$$c(x_i) = i,$$

$$c(V(H_i)) = \begin{cases} [1, m+2] - \{i, i+1\} & \text{if } i \neq m+2, \\ [1, m+2] - \{1, i\} & \text{otherwise.} \end{cases}$$

Now, we will show that $c$ is a locating coloring on $K_n \odot K_m$. If $d(u, x_1) = d(v, x_1)$ then we have: (i) $u = x_k$ and $v = x_l$ for some $k, l$; (ii) $u = x_i$ and $v = a_{1k}$ for some $i \neq 1$ and $k$, or (iii) $u = a_{it}$ and $v = a_{js}$ for some $i, j, t, s$. If (i) holds then $u$ and $v$ must be in different classes. If (ii) holds then $d(u, S_2) \neq d(v, S_2)$. If (iii) holds then $d(u, S_{i+1}) \neq d(v, S_{i+1})$ or $d(u, S_1) \neq d(v, S_1)$. Therefore, $c$ is a locating coloring and so $\chi_L(K_n \odot K_m) = m+2$ for $2 \leq n \leq m+2$.

Next, if $n > m+2$ then it is clear that $\chi_L(K_n \odot K_m) \geq n$. Now, let us define a mapping $c$ such that $c(x_i) = i$, and $c(V(H_i)) = X_i$, where $X_i$ is any $m$-subset of $\{1, 2, \cdots, n\}$ not containing $i$ and $i+1 \mod n$, for each $i$. It can be verified that $c$ is a locating coloring. Therefore $\chi_L(K_n \odot K_m) = n$. □

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