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THE DETERMINANT OF THE ADJACENCY MATRIX OF CYCLE-POWER GRAPH C_n^4

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Abstract

Cycle-Power Graphs, C_n^d is a graph that has n vertices and two vertices u and v are adjacent if and only if distance between u and v not greater than d. In this paper, we show that the determinant of the adjacency matrix of cycle-power graph C_n^4 are as follows

$$det(A(C_n^4)) = \begin{cases} 0 & ; n \equiv 0, 2, 4, 6, 8 \bmod 10 \\ 8 & ; n \equiv 1, 3, 7, 9 \bmod 10 \\ 128 & ; n \equiv 5 \bmod 10. \end{cases}$$

and the condition for the adjacency matrix of cycle-power graph C_n^d is singular matrix.

1 Introduction

Let G be a simple graph with vertex set $V(G) = \{v_1, v_2, ..., v_n\}$ and edge set $E(G)$. The adjacency matrix $A(G)$ of G is the matrix $[a_{ij}]_{n \times n}$ where

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$$a_{ij} = \begin{cases} 1 & ; \text{if } v_i \text{ and } v_j \text{ are adjacent} \\ 0 & ; \text{otherwise.} \end{cases}$$

The determinant of adjacency matrix of G is $det(A(G)$ and k^{th} eigenvalues of the adjacency matrix is $E(G;k)$ which are independent of the choice of vertices in adjacency matrix and are an invariant of G, respectively.

In 1974 and In 1980, N. Biggs [3] and D.M. Cvetkovic et.al [5] have published about determinant of adjacency matrix of some graphs, such as K_n, C_n, P_n and W_n. In 2002, M. Doob [6] had found determinant of adjacency matrix of graph $G(r,t)$. In 2009, A. Abdollahi [1] had found determinant of adjacency matrix of graphs. In 2011, B. Gyurov and J. Cloud [8] had found determinant of adjacency matrix of Pin-wheel graph. In 2014, N. Adsawatithisakul and D. Samana [2] had shown determinant of adjacency matrix of square cycle graph. Furthermore, there are studies of graph about some properties of determinant for example, S. Hu [10] and A. Abdollahi [1] have found that determinant of graph with exactly one cycle and exactly two cycle, respectively.

Cycle-power, C_n^d is a graph that has n vertices and distance each pair of vertex is less or equal d . For example,

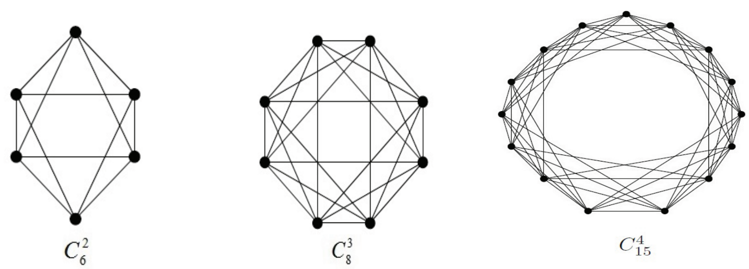

Figure 1: some cycle-power graphs C_n^d

Moreover, there are studies about cycle power graph such as, In[4] and [11] studied about the colouring in cycle power, Y.Hoa, C.Woo and P.Chen [9] construct the sandpile group in cycle power , D.Li and M.Liu [12] consider cycle power and their complements which satisfy Hadwiger's conjecture.

In this article, we focus $d = 4$ of cycle-power graph which determinant of cycle-power C_n^4 can find by Proposition 1.1 and Theorem 1.2.

Proposition 1.1. *[3] Suppose that $[0, a_2, ..., a_n]$ is the first row of the adjacency matrix of a circulant graph G. Then the eigenvalues of graph G is denoted $E(G;k)$,*

$$E(G;k) = \sum_{j=1}^{n} a_j z^{j-1}$$

where $z = e^{\frac{2k\pi i}{n}}, k = 1, 2, ..., n$.

Cycle-power graph C_n^4 is a circulant graph then eigenvalues of C_n^4 is

$$E(C_n^4; k) = z + z^2 + z^3 + z^4 + z^{n-4} + z^{n-3} + z^{n-2} + z^{n-1}. \tag{1.1}$$

Determinant of a square matrix can be find by eigenvalue its as below.

Theorem 1.2. *[7] Let* $\lambda_1, ..., \lambda_n$ *be a eigenvalues of a square matrix* A. *Then*

$$det(A) = \prod_{i=1}^{n} \lambda_i.$$

Next, we present lemma that will be used in the proof of determinant of adjacency matrix of cycle-power graph C_n^4.

2 Main Results

In this section, we proof about lemma and main theorem as follows.

Lemma 2.1. *Let* n *be an odd integer with* $n \geq 10$ *and* $n \not\equiv 5 \bmod 10$. *Then*

$$\prod_{k=1}^{n-1} (\cos\frac{k\pi}{n} \cos\frac{2k\pi}{n} \cos\frac{5k\pi}{n}) = 8^{-(n-1)}.$$

Proof. It can be proved by

$$\begin{aligned}
\prod_{k=1}^{n-1} (\cos\frac{k\pi}{n} \cos\frac{2k\pi}{n} \cos\frac{5k\pi}{n}) &= \prod_{k=1}^{n-1} \frac{\sin\frac{2k\pi}{n} \sin\frac{4k\pi}{n} \sin\frac{10k\pi}{n}}{2\sin\frac{k\pi}{n} 2\sin\frac{2k\pi}{n} 2\sin\frac{5k\pi}{n}} \\
&= \frac{1}{8^{n-1}} \left(\frac{\prod_{k=\frac{n+1}{2}}^{n-1} \sin\frac{2k\pi}{n} \sin\frac{4k\pi}{n} \sin\frac{10k\pi}{n}}{\prod_{k=1}^{\frac{n-1}{2}} \sin\frac{(2k-1)\pi}{n} \sin\frac{(4k-2)\pi}{n} \sin\frac{(10k-5)\pi}{n}} \right) \\
&= \frac{1}{8^{n-1}} \left(\frac{\prod_{k=1}^{\frac{n-1}{2}} \sin\frac{(2k-1)\pi}{n} \sin\frac{(4k-2)\pi}{n} \sin\frac{(10k-5)\pi}{n}}{\prod_{k=1}^{\frac{n-1}{2}} \sin\frac{(2k-1)\pi}{n} \sin\frac{(4k-2)\pi}{n} \sin\frac{(10k-5)\pi}{n}} \right) \\
&= 8^{-(n-1)}.
\end{aligned}$$

□

Lemma 2.2. *Let* q *be a positive number. Then*

$$\begin{aligned}
&\prod_{k=1}^{2q} (\cos\frac{k\pi}{10q+5} \cos\frac{2k\pi}{10q+5} \cos\frac{5k\pi}{10q+5}) \prod_{k=2q+2}^{4q+1} (\cos\frac{k\pi}{10q+5} \cos\frac{2k\pi}{10q+5} \cos\frac{5k\pi}{10q+5}) \\
&\prod_{k=4q+3}^{6q+2} (\cos\frac{k\pi}{10q+5} \cos\frac{2k\pi}{10q+5} \cos\frac{5k\pi}{10q+5}) \prod_{k=6q+4}^{8q+3} (\cos\frac{k\pi}{10q+5} \cos\frac{2k\pi}{10q+5} \cos\frac{5k\pi}{10q+5}) \\
&\prod_{k=8q+5}^{10q+4} (\cos\frac{k\pi}{10q+5} \cos\frac{2k\pi}{10q+5} \cos\frac{5k\pi}{10q+5}) = 8^{-10q}
\end{aligned} \tag{2.1}$$

Proof. The left hand side of (2.1) is

$$\prod_{k=1}^{2q}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})\prod_{k=2q+2}^{4q+1}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$\prod_{k=4q+3}^{6q+2}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})\prod_{k=6q+4}^{8q+3}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$\prod_{k=8q+5}^{10q+4}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$=\left(\prod_{k=1}^{2q}\frac{\sin\frac{2k\pi}{10q+5}\sin\frac{4k\pi}{10q+5}\sin\frac{10k\pi}{10q+5}}{2\sin\frac{k\pi}{10q+5}2\sin\frac{2k\pi}{10q+5}2\sin\frac{5k\pi}{10q+5}}\right)\left(\prod_{k=2q+2}^{4q+1}\frac{\sin\frac{2k\pi}{10q+5}\sin\frac{4k\pi}{10q+5}\sin\frac{10k\pi}{10q+5}}{2\sin\frac{k\pi}{10q+5}2\sin\frac{2k\pi}{10q+5}2\sin\frac{5k\pi}{10q+5}}\right)$$
$$\left(\prod_{k=4q+3}^{6q+2}\frac{\sin\frac{2k\pi}{10q+5}\sin\frac{4k\pi}{10q+5}\sin\frac{10k\pi}{10q+5}}{2\sin\frac{k\pi}{10q+5}2\sin\frac{2k\pi}{10q+5}2\sin\frac{5k\pi}{10q+5}}\right)\left(\prod_{k=6q+4}^{8q+3}\frac{\sin\frac{2k\pi}{10q+5}\sin\frac{4k\pi}{10q+5}\sin\frac{10k\pi}{10q+5}}{2\sin\frac{k\pi}{10q+5}2\sin\frac{2k\pi}{10q+5}2\sin\frac{5k\pi}{10q+5}}\right)$$
$$\left(\prod_{k=8q+5}^{10q+4}\frac{\sin\frac{2k\pi}{10q+5}\sin\frac{4k\pi}{10q+5}\sin\frac{10k\pi}{10q+5}}{2\sin\frac{k\pi}{10q+5}2\sin\frac{2k\pi}{10q+5}2\sin\frac{5k\pi}{10q+5}}\right)$$
$$=\frac{1}{8^{10q}}\left(\frac{\prod_{k=1}^{q}\sin\frac{(2k-1)\pi}{10q+5}\sin\frac{(4k-2)\pi}{10q+5}\sin\frac{(10k-5)\pi}{10q+5}}{\prod_{k=1}^{q}\sin\frac{(2k-1)\pi}{10q+5}\sin\frac{(4k-2)\pi}{10q+5}\sin\frac{(10k-5)\pi}{10q+5}}\right)$$
$$\left(\frac{\prod_{k=2q+2}^{3q+1}\sin\frac{(2k-(2q+1))\pi}{10q+5}\sin\frac{(4k-(4q+2))\pi}{10q+5}\sin\frac{(10k-(10q+5))\pi}{10q+5}}{\prod_{k=2q+2}^{3q+1}\sin\frac{(2k-(2q+1))\pi}{10q+5}\sin\frac{(4k-(4q+2))\pi}{10q+5}\sin\frac{(10k-(10q+5))\pi}{10q+5}}\right)$$
$$\left(\frac{\prod_{k=4q+3}^{5q+2}\sin\frac{(2k-(4q+3))\pi}{10q+5}\sin\frac{(4k-(8q+6))\pi}{10q+5}\sin\frac{(10k-(20q+15))\pi}{10q+5}}{\prod_{k=4q+3}^{5q+2}\sin\frac{(2k-(4q+3))\pi}{10q+5}\sin\frac{(4k-(8q+6))\pi}{10q+5}\sin\frac{(10k-(20q+15))\pi}{10q+5}}\right)$$
$$\left(\frac{\prod_{k=6q+4}^{7q+3}\sin\frac{(2k-(6q+3))\pi}{10q+5}\sin\frac{(4k-(12q+6))\pi}{10q+5}\sin\frac{(10k-(30q+15))\pi}{10q+5}}{\prod_{k=6q+4}^{7q+3}\sin\frac{(2k-(6q+3))\pi}{10q+5}\sin\frac{(4k-(12q+6))\pi}{10q+5}\sin\frac{(10k-(30q+15))\pi}{10q+5}}\right)$$
$$\left(\frac{\prod_{k=8q+5}^{9q+4}\sin\frac{(2k-(8q+5))\pi}{10q+5}\sin\frac{(4k-(16q+10))\pi}{10q+5}\sin\frac{(10k-(40q+25))\pi}{10q+5}}{\prod_{k=8q+5}^{9q+4}\sin\frac{(2k-(8q+5))\pi}{10q+5}\sin\frac{(4k-(16q+10))\pi}{10q+5}\sin\frac{(10k-(40q+25))\pi}{10q+5}}\right)$$
$$=8^{-10q}$$

□

Next, we use Lemma 2.1 and 2.2 to find determinant of adjacency matrix of cycle-power graph C_n^4.

Theorem 2.3. *Let C_n^4 be a cycle-power graph with n vertices. Then*

$$det(A(C_n^4)) = \begin{cases} 0 & ; n \equiv 0,2,4,6,8 \bmod 10 \\ 8 & ; n \equiv 1,3,7,9 \bmod 10 \\ 128 & ; n \equiv 5 \bmod 10. \end{cases}$$

Proof. Let $E(C_n^4;k)$ be a k^{th} eigenvalue of adjacency matrix of cycle-power graph C_n^4. From (1.1), We get

$$\begin{aligned} E(C_n^4;k) &= e^{\frac{2k\pi i}{n}} + e^{\frac{4k\pi i}{n}} + e^{\frac{6k\pi i}{n}} + e^{\frac{8k\pi i}{n}} + e^{\frac{2k(n-4)\pi i}{n}} + e^{\frac{2k(n-3)\pi i}{n}} + e^{\frac{2k(n-2)\pi i}{n}} + e^{\frac{2k(n-1)\pi i}{n}} \\ &= e^{\frac{2k\pi i}{n}} + e^{\frac{4k\pi i}{n}} + e^{\frac{6k\pi i}{n}} + e^{\frac{8k\pi i}{n}} + e^{\frac{2kn\pi i}{n}} \cdot e^{\frac{-8k\pi i}{n}} + e^{\frac{2kn\pi i}{n}} \cdot e^{\frac{-6k\pi i}{n}} + e^{\frac{2kn\pi i}{n}} \cdot e^{\frac{-4k\pi i}{n}} \\ &\quad + e^{\frac{2kn\pi i}{n}} \cdot e^{\frac{-2k\pi i}{n}}. \end{aligned}$$

By Euler's formula, we obtain

$$\begin{aligned} E(C_n^4;k) = (\cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}) + (\cos\frac{4k\pi}{n} + i\sin\frac{4k\pi}{n}) \\ + (\cos\frac{6k\pi}{n} + i\sin\frac{6k\pi}{n}) + (\cos\frac{8k\pi}{n} + i\sin\frac{8k\pi}{n}) \\ + (\cos\frac{-8k\pi}{n} + i\sin\frac{-8k\pi}{n}) + (\cos\frac{-6k\pi}{n} + i\sin\frac{-6k\pi}{n}) \\ + (\cos\frac{-4k\pi}{n} + i\sin\frac{-4k\pi}{n}) + (\cos\frac{-2k\pi}{n} + i\sin\frac{-2k\pi}{n}) \\ = 2\cos\frac{2k\pi}{n} + 2\cos\frac{4k\pi}{n} + 2\cos\frac{6k\pi}{n} + 2\cos\frac{8k\pi}{n}. \end{aligned}$$

We can rewrite

$$E(C_n^4;k) = 8(\cos\frac{k\pi}{n}\cos\frac{2k\pi}{n}\cos\frac{5k\pi}{n}). \tag{2.2}$$

From (2.2), we have

$$\begin{aligned} det(A(C_n^4)) &= \prod_{k=1}^{n} E(C_n^4;k) \\ &= \prod_{k=1}^{n} 8(\cos\frac{k\pi}{n}\cos\frac{2k\pi}{n}\cos\frac{5k\pi}{n}). \end{aligned} \tag{2.3}$$

Consider n as follows,

Case I, $n \equiv 0,2,4,6,8 \bmod 10$. Since $1 \leq k \leq n$ and consider (2.2) when $k = \frac{n}{2}$. Then

$$\begin{aligned} E(C_n^4;\frac{n}{2}) &= 8(\cos\frac{\pi}{2}\cos\pi\cos\frac{5\pi}{2}) \\ &= 0. \end{aligned}$$

From (2.3), we obtain

$$\begin{aligned}
det(A(C_n^4)) &= \prod_{k=1}^{n} E(C_n^4; k) \\
&= 0.
\end{aligned}$$

Therefore, $det(A(C_n^4)) = 0$ when $n \equiv 0, 2, 4, 6, 8 \bmod 10$.
Case II, $n \equiv 1, 3, 7, 9 \bmod 10$. Since n is odd and $n \not\equiv 5 \bmod 10$. Then

$$\begin{aligned}
det(A(C_n^4)) &= \prod_{k=1}^{n} E(C_n^4; k) \\
&= \prod_{k=1}^{n} 8(\cos\frac{k\pi}{n}\cos\frac{2k\pi}{n}\cos\frac{5k\pi}{n}) \\
&= 8(\cos\frac{n\pi}{n}\cos\frac{2n\pi}{n}\cos\frac{5n\pi}{n})8^{n-1}\prod_{k=1}^{n-1}(\cos\frac{k\pi}{n}\cos\frac{2k\pi}{n}\cos\frac{5k\pi}{n})
\end{aligned}$$

Using Lemma 2.1, we have

$$\begin{aligned}
det(A(C_n^4)) &= \prod_{k=1}^{n} E(C_n^4; k) \\
&= 8 \cdot (8^{n-1}) \cdot (8^{-(n-1)}) \\
&= 8
\end{aligned}$$

Therefore $det(A(C_n^4)) = 8$ when n is odd and $n \not\equiv 5 \bmod 10$.
Case III, $n \equiv 5 \bmod 10$. Then $n = 10q + 5$, $\exists q \in \mathbb{Z}^+$.
From (2.3), we obtain

$$\begin{aligned}
det(A(C_n^4)) &= \prod_{k=1}^{n} E(C_n^4; k) \\
&= \prod_{k=1}^{10q+5} 8(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5}) \\
&= 8(\cos\frac{(2q+1)\pi}{10q+5}\cos\frac{2(2q+1)\pi}{10q+5}\cos\frac{5(2q+1)\pi}{10q+5}) \\
&\quad 8(\cos\frac{(4q+2)\pi}{10q+5}\cos\frac{2(4q+2)\pi}{10q+5}\cos\frac{5(4q+2)\pi}{10q+5}) \\
&\quad 8(\cos\frac{(6q+3)\pi}{10q+5}\cos\frac{2(6q+3)\pi}{10q+5}\cos\frac{5(6q+3)\pi}{10q+5}) \\
&\quad 8(\cos\frac{(8q+4)\pi}{10q+5}\cos\frac{2(8q+4)\pi}{10q+5}\cos\frac{5(8q+4)\pi}{10q+5}) \\
&\quad 8(\cos\frac{(10q+5)\pi}{10q+5}\cos\frac{2(10q+5)\pi}{10q+5}\cos\frac{5(10q+5)\pi}{10q+5})
\end{aligned}$$

$$8^{10q}\prod_{k=1}^{2q}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$\prod_{k=2q+2}^{4q+1}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$\prod_{k=4q+3}^{6q+2}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$\prod_{k=6q+4}^{8q+3}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$\prod_{k=8q+5}^{10q+4}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$=(-2)(-2)(-2)(-2)(8)(8^{10q})\prod_{k=1}^{2q}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$\prod_{k=2q+2}^{4q+1}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$\prod_{k=4q+3}^{6q+2}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$\prod_{k=6q+4}^{8q+3}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$
$$\prod_{k=8q+5}^{10q+4}(\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5}).$$

Using Lemma 2.2, we have

$$\begin{aligned} det(A(C_n^4)) &= (-2)(-2)(-2)(-2)(8)(8^{10q})(8^{-10q}) \\ &= 128 \end{aligned}$$

Therefore $det(A(C_n^4)) = 128$ when $n \equiv 5 \bmod 10$. From case I, II and III, then

$$det(A(C_n^4)) = \begin{cases} 0 & ; n \equiv 0,2,4,6,8 \bmod 10 \\ 8 & ; n \equiv 1,3,7,9 \bmod 10 \\ 128 & ; n \equiv 5 \bmod 10. \end{cases}$$

□

Moreover, observe that $det(A(C_n^d)) = 0$ where $n = 2 + 2d$ and d are even, we have Corollary 2.4.

Corollary 2.4. *If $n = 2 + 2d$ and d is even then the adjacency matrix of cycle-power graph C_n^d is singular .*

Proof. Let d be even. By proposition 1.1, we have

$$\begin{aligned} E(C_n^d; k) &= z + z^2 + \ldots + z^d + z^{n-d} + \ldots + z^{n-2} + z^{n-1} \\ &= 2\cos\frac{2k\pi}{n} + 2\cos\frac{4k\pi}{n} + \ldots + 2\cos\frac{2kd\pi}{n} \\ &= 2\sum_{j=1}^{d}\cos\frac{2jk\pi}{n} \end{aligned} \tag{5}$$

Since $n = 2 + 2d$ and $1 \leq k \leq n$. Consider (5) when $k = \frac{n}{2}$. Then

$$\begin{aligned} E(C_n^d; \frac{n}{2}) &= 2\cos\frac{2(\frac{n}{2})\pi}{n} + 2\cos\frac{4(\frac{n}{2})\pi}{n} + \ldots + 2\cos\frac{2(\frac{n}{2})d\pi}{n} \\ &= 0. \end{aligned}$$

Therefore the adjacency matrix of C_n^d is singular matrix. □

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