THE DETERMINANT OF THE ADJACENCY MATRIX OF CYCLE-POWER GRAPH C_n^4

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Abstract

Cycle-Power Graphs, C_n^d is a graph that has n vertices and two vertices u and v are adjacent if and only if distance between u and v not greater than d. In this paper, we show that the determinant of the adjacency matrix of cycle-power graph C_n^4 are as follows

$$det(A(C_n^4)) = \begin{cases} 0 & ; n \equiv 0, 2, 4, 6, 8 \bmod 10 \\ 8 & ; n \equiv 1, 3, 7, 9 \bmod 10 \\ 128 & ; n \equiv 5 \bmod 10. \end{cases}$$

and the condition for the adjacency matrix of cycle-power graph C_n^d is singular matrix.

1 Introduction

Let G be a simple graph with vertex set $V(G) = \{v_1, v_2, ..., v_n\}$ and edge set $E(G)$. The adjacency matrix $A(G)$ of G is the matrix $[a_{ij}]_{n \times n}$ where

$$a_{ij} = \begin{cases} 1 & \text{; if } v_i \text{ and } v_j \text{ are adjacent} \\ 0 & \text{; otherwise.} \end{cases}$$

The determinant of adjacency matrix of G is $det(A(G))$ and k^{th} eigenvalues of the adjacency matrix is $E(G; k)$ which are independent of the choice of vertices in adjacency matrix and are an invariant of G, respectively.

In 1974 and In 1980, N. Biggs [3] and D.M. Cvetkovic et.al [5] have published about determinant of adjacency matrix of some graphs, such as K_n, C_n, P_n and W_n. In 2002, M. Doob [6] had found determinant of adjacency matrix of graph $G(r, t)$. In 2009, A. Abdollahi [1] had found determinant of adjacency matrix of graphs. In 2011, B. Gyurov and J. Cloud [8] had found determinant of adjacency matrix of Pin-wheel graph. In 2014, N. Adsawatithisakul and D. Samana [2] had shown determinant of adjacency matrix of square cycle graph. Furthermore, there are studies of graph about some properties of determinant for example, S. Hu [10] and A. Abdollahi [1] have found that determinant of graph with exactly one cycle and exactly two cycle, respectively.

Cycle-power, C_n^d is a graph that has n vertices and distance each pair of vertex is less or equal d . For example,

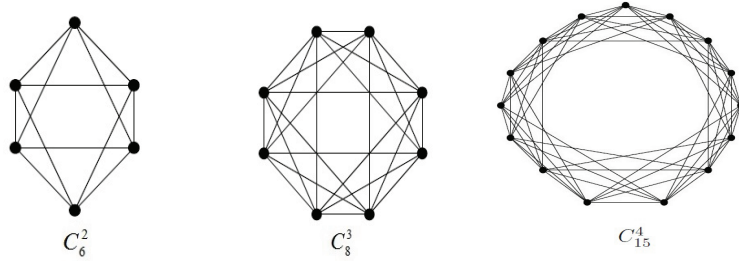$$C_6^2 \qquad\qquad C_8^3 \qquad\qquad C_{15}^4$$

Figure 1: some cycle-power graphs C_n^d

Moreover, there are studies about cycle power graph such as, In[4] and [11] studied about the colouring in cycle power, Y.Hoa, C.Woo and P.Chen [9] construct the sandpile group in cycle power , D.Li and M.Liu [12] consider cycle power and their complements which satisfy Hadwiger's conjecture.

In this article, we focus $d = 4$ of cycle-power graph which determinant of cycle-power C_n^4 can find by Proposition 1.1 and Theorem 1.2.

Proposition 1.1. *[3] Suppose that $[0, a_2, ..., a_n]$ is the first row of the adjacency matrix of a circulant graph G. Then the eigenvalues of graph G is denoted $E(G; k)$,*

$$E(G; k) = \sum_{j=1}^{n} a_j z^{j-1}$$

where $z = e^{\frac{2k\pi i}{n}}, k = 1, 2, ..., n$.

Cycle-power graph C_n^4 is a circulant graph then eigenvalues of C_n^4 is

$$E(C_n^4; k) = z + z^2 + z^3 + z^4 + z^{n-4} + z^{n-3} + z^{n-2} + z^{n-1}. \qquad (1.1)$$

Determinant of a square matrix can be find by eigenvalue its as below.

Theorem 1.2. *[7] Let $\lambda_1, ..., \lambda_n$ be a eigenvalues of a square matrix A. Then*

$$det(A) = \prod_{i=1}^{n} \lambda_i.$$

Next, we present lemma that will be used in the proof of determinant of adjacency matrix of cycle-power graph C_n^4.

2   Main Results

In this section, we proof about lemma and main theorem as follows.

Lemma 2.1. *Let n be an odd integer with $n \geq 10$ and $n \not\equiv 5$ mod 10. Then*

$$\prod_{k=1}^{n-1} (\cos \frac{k\pi}{n} \cos \frac{2k\pi}{n} \cos \frac{5k\pi}{n}) = 8^{-(n-1)}.$$

Proof. It can be proved by

$$\prod_{k=1}^{n-1} (\cos \frac{k\pi}{n} \cos \frac{2k\pi}{n} \cos \frac{5k\pi}{n}) = \prod_{k=1}^{n-1} \frac{\sin \frac{2k\pi}{n} \sin \frac{4k\pi}{n} \sin \frac{10k\pi}{n}}{2 \sin \frac{k\pi}{n} 2 \sin \frac{2k\pi}{n} 2 \sin \frac{5k\pi}{n}}$$

$$= \frac{1}{8^{n-1}} \left(\frac{\prod_{k=\frac{n+1}{2}}^{n-1} \sin \frac{2k\pi}{n} \sin \frac{4k\pi}{n} \sin \frac{10k\pi}{n}}{\prod_{k=1}^{\frac{n-1}{2}} \sin \frac{(2k-1)\pi}{n} \sin \frac{(4k-2)\pi}{n} \sin \frac{(10k-5)\pi}{n}} \right)$$

$$= \frac{1}{8^{n-1}} \left(\frac{\prod_{k=1}^{\frac{n-1}{2}} \sin \frac{(2k-1)\pi}{n} \sin \frac{(4k-2)\pi}{n} \sin \frac{(10k-5)\pi}{n}}{\prod_{k=1}^{\frac{n-1}{2}} \sin \frac{(2k-1)\pi}{n} \sin \frac{(4k-2)\pi}{n} \sin \frac{(10k-5)\pi}{n}} \right)$$

$$= 8^{-(n-1)}.$$

$\square$

Lemma 2.2. *Let q be a positive number. Then*

$$\prod_{k=1}^{2q} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5}) \prod_{k=2q+2}^{4q+1} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$\prod_{k=4q+3}^{6q+2} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5}) \prod_{k=6q+4}^{8q+3} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$\prod_{k=8q+5}^{10q+4} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5}) = 8^{-10q}$$

$$(2.1)$$

Proof. The left hand side of (2.1) is

$$\prod_{k=1}^{2q} (\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5}) \prod_{k=2q+2}^{4q+1} (\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$

$$\prod_{k=4q+3}^{6q+2} (\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5}) \prod_{k=6q+4}^{8q+3} (\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$

$$\prod_{k=8q+5}^{10q+4} (\cos\frac{k\pi}{10q+5}\cos\frac{2k\pi}{10q+5}\cos\frac{5k\pi}{10q+5})$$

$$= \left(\prod_{k=1}^{2q}\frac{\sin\frac{2k\pi}{10q+5}\sin\frac{4k\pi}{10q+5}\sin\frac{10k\pi}{10q+5}}{2\sin\frac{k\pi}{10q+5}2\sin\frac{2k\pi}{10q+5}2\sin\frac{5k\pi}{10q+5}}\right) \left(\prod_{k=2q+2}^{4q+1}\frac{\sin\frac{2k\pi}{10q+5}\sin\frac{4k\pi}{10q+5}\sin\frac{10k\pi}{10q+5}}{2\sin\frac{k\pi}{10q+5}2\sin\frac{2k\pi}{10q+5}2\sin\frac{5k\pi}{10q+5}}\right)$$

$$\left(\prod_{k=4q+3}^{6q+2}\frac{\sin\frac{2k\pi}{10q+5}\sin\frac{4k\pi}{10q+5}\sin\frac{10k\pi}{10q+5}}{2\sin\frac{k\pi}{10q+5}2\sin\frac{2k\pi}{10q+5}2\sin\frac{5k\pi}{10q+5}}\right) \left(\prod_{k=6q+4}^{8q+3}\frac{\sin\frac{2k\pi}{10q+5}\sin\frac{4k\pi}{10q+5}\sin\frac{10k\pi}{10q+5}}{2\sin\frac{k\pi}{10q+5}2\sin\frac{2k\pi}{10q+5}2\sin\frac{5k\pi}{10q+5}}\right)$$

$$\left(\prod_{k=8q+5}^{10q+4}\frac{\sin\frac{2k\pi}{10q+5}\sin\frac{4k\pi}{10q+5}\sin\frac{10k\pi}{10q+5}}{2\sin\frac{k\pi}{10q+5}2\sin\frac{2k\pi}{10q+5}2\sin\frac{5k\pi}{10q+5}}\right)$$

$$= \frac{1}{8^{10q}} \left(\frac{\prod_{k=1}^{q}\sin\frac{(2k-1)\pi}{10q+5}\sin\frac{(4k-2)\pi}{10q+5}\sin\frac{(10k-5)\pi}{10q+5}}{\prod_{k=1}^{q}\sin\frac{(2k-1)\pi}{10q+5}\sin\frac{(4k-2)\pi}{10q+5}\sin\frac{(10k-5)\pi}{10q+5}}\right)$$

$$\left(\frac{\prod_{k=2q+2}^{3q+1}\sin\frac{(2k-(2q+1))\pi}{10q+5}\sin\frac{(4k-(4q+2))\pi}{10q+5}\sin\frac{(10k-(10q+5))\pi}{10q+5}}{\prod_{k=2q+2}^{3q+1}\sin\frac{(2k-(2q+1))\pi}{10q+5}\sin\frac{(4k-(4q+2))\pi}{10q+5}\sin\frac{(10k-(10q+5))\pi}{10q+5}}\right)$$

$$\left(\frac{\prod_{k=4q+3}^{5q+2}\sin\frac{(2k-(4q+3))\pi}{10q+5}\sin\frac{(4k-(8q+6))\pi}{10q+5}\sin\frac{(10k-(20q+15))\pi}{10q+5}}{\prod_{k=4q+3}^{5q+2}\sin\frac{(2k-(4q+3))\pi}{10q+5}\sin\frac{(4k-(8q+6))\pi}{10q+5}\sin\frac{(10k-(20q+15))\pi}{10q+5}}\right)$$

$$\left(\frac{\prod_{k=6q+4}^{7q+3}\sin\frac{(2k-(6q+3))\pi}{10q+5}\sin\frac{(4k-(12q+6))\pi}{10q+5}\sin\frac{(10k-(30q+15))\pi}{10q+5}}{\prod_{k=6q+4}^{7q+3}\sin\frac{(2k-(6q+3))\pi}{10q+5}\sin\frac{(4k-(12q+6))\pi}{10q+5}\sin\frac{(10k-(30q+15))\pi}{10q+5}}\right)$$

$$\left(\frac{\prod_{k=8q+5}^{9q+4}\sin\frac{(2k-(8q+5))\pi}{10q+5}\sin\frac{(4k-(16q+10))\pi}{10q+5}\sin\frac{(10k-(40q+25))\pi}{10q+5}}{\prod_{k=8q+5}^{9q+4}\sin\frac{(2k-(8q+5))\pi}{10q+5}\sin\frac{(4k-(16q+10))\pi}{10q+5}\sin\frac{(10k-(40q+25))\pi}{10q+5}}\right)$$

$$= 8^{-10q}$$

<div style="text-align: right">□</div>

Next, we use Lemma 2.1 and 2.2 to find determinant of adjacency matrix of cycle-power graph C_n^4.

Theorem 2.3. *Let C_n^4 be a cycle-power graph with n vertices. Then*

$$det(A(C_n^4)) = \begin{cases} 0 & ; n \equiv 0, 2, 4, 6, 8 \bmod 10 \\ 8 & ; n \equiv 1, 3, 7, 9 \bmod 10 \\ 128 & ; n \equiv 5 \bmod 10. \end{cases}$$

Proof. Let $E(C_n^4; k)$ be a kth eigenvalue of adjacency matrix of cycle-power graph C_n^4. From (1.1), We get

$$E(C_n^4; k) = e^{\frac{2k\pi i}{n}} + e^{\frac{4k\pi i}{n}} + e^{\frac{6k\pi i}{n}} + e^{\frac{8k\pi i}{n}} + e^{\frac{2k(n-4)\pi i}{n}} + e^{\frac{2k(n-3)\pi i}{n}} + e^{\frac{2k(n-2)\pi i}{n}} + e^{\frac{2k(n-1)\pi i}{n}}$$

$$= e^{\frac{2k\pi i}{n}} + e^{\frac{4k\pi i}{n}} + e^{\frac{6k\pi i}{n}} + e^{\frac{8k\pi i}{n}} + e^{\frac{2kn\pi i}{n}} \cdot e^{\frac{-8k\pi i}{n}} + e^{\frac{2kn\pi i}{n}} \cdot e^{\frac{-6k\pi i}{n}} + e^{\frac{2kn\pi i}{n}} \cdot e^{\frac{-4k\pi i}{n}}$$

$$+ e^{\frac{2kn\pi i}{n}} \cdot e^{\frac{-2k\pi i}{n}}.$$

By Euler's formula, we obtain

$$E(C_n^4; k) = (\cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n}) + (\cos \frac{4k\pi}{n} + i \sin \frac{4k\pi}{n})$$
$$+ (\cos \frac{6k\pi}{n} + i \sin \frac{6k\pi}{n}) + (\cos \frac{8k\pi}{n} + i \sin \frac{8k\pi}{n})$$
$$+ (\cos \frac{-8k\pi}{n} + i \sin \frac{-8k\pi}{n}) + (\cos \frac{-6k\pi}{n} + i \sin \frac{-6k\pi}{n})$$
$$+ (\cos \frac{-4k\pi}{n} + i \sin \frac{-4k\pi}{n}) + (\cos \frac{-2k\pi}{n} + i \sin \frac{-2k\pi}{n})$$
$$= 2 \cos \frac{2k\pi}{n} + 2 \cos \frac{4k\pi}{n} + 2 \cos \frac{6k\pi}{n} + 2 \cos \frac{8k\pi}{n}.$$

We can rewrite
$$E(C_n^4; k) = 8(\cos \frac{k\pi}{n} \cos \frac{2k\pi}{n} \cos \frac{5k\pi}{n}). \tag{2.2}$$

From (2.2), we have

$$det(A(C_n^4)) = \prod_{k=1}^{n} E(C_n^4; k)$$
$$= \prod_{k=1}^{n} 8(\cos \frac{k\pi}{n} \cos \frac{2k\pi}{n} \cos \frac{5k\pi}{n}). \tag{2.3}$$

Consider n as follows,
Case I, $n \equiv 0, 2, 4, 6, 8 \bmod 10$. Since $1 \leq k \leq n$ and consider (2.2) when $k = \frac{n}{2}$. Then

$$E(C_n^4; \frac{n}{2}) = 8(\cos \frac{\pi}{2} \cos \pi \cos \frac{5\pi}{2})$$
$$= 0.$$

From (2.3), we obtain

$$det(A(C_n^4)) = \prod_{k=1}^{n} E(C_n^4; k)$$
$$= 0.$$

Therefore, $det(A(C_n^4)) = 0$ when $n \equiv 0, 2, 4, 6, 8 \bmod 10$.

Case II, $n \equiv 1, 3, 7, 9 \bmod 10$. Since n is odd and $n \not\equiv 5 \bmod 10$. Then

$$det(A(C_n^4)) = \prod_{k=1}^{n} E(C_n^4; k)$$
$$= \prod_{k=1}^{n} 8(\cos \frac{k\pi}{n} \cos \frac{2k\pi}{n} \cos \frac{5k\pi}{n})$$
$$= 8(\cos \frac{n\pi}{n} \cos \frac{2n\pi}{n} \cos \frac{5n\pi}{n}) 8^{n-1} \prod_{k=1}^{n-1} (\cos \frac{k\pi}{n} \cos \frac{2k\pi}{n} \cos \frac{5k\pi}{n})$$

Using Lemma 2.1, we have

$$det(A(C_n^4)) = \prod_{k=1}^{n} E(C_n^4; k)$$
$$= 8 \cdot (8^{n-1}) \cdot (8^{-(n-1)})$$
$$= 8$$

Therefore $det(A(C_n^4)) = 8$ when n is odd and $n \not\equiv 5 \bmod 10$.

Case III, $n \equiv 5 \bmod 10$. Then $n = 10q + 5$, $\exists q \in \mathbb{Z}^+$.

From (2.3), we obtain

$$det(A(C_n^4)) = \prod_{k=1}^{n} E(C_n^4; k)$$
$$= \prod_{k=1}^{10q+5} 8(\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$
$$= 8(\cos \frac{(2q+1)\pi}{10q+5} \cos \frac{2(2q+1)\pi}{10q+5} \cos \frac{5(2q+1)\pi}{10q+5})$$
$$8(\cos \frac{(4q+2)\pi}{10q+5} \cos \frac{2(4q+2)\pi}{10q+5} \cos \frac{5(4q+2)\pi}{10q+5})$$
$$8(\cos \frac{(6q+3)\pi}{10q+5} \cos \frac{2(6q+3)\pi}{10q+5} \cos \frac{5(6q+3)\pi}{10q+5})$$
$$8(\cos \frac{(8q+4)\pi}{10q+5} \cos \frac{2(8q+4)\pi}{10q+5} \cos \frac{5(8q+4)\pi}{10q+5})$$
$$8(\cos \frac{(10q+5)\pi}{10q+5} \cos \frac{2(10q+5)\pi}{10q+5} \cos \frac{5(10q+5)\pi}{10q+5})$$

$$8^{10q} \prod_{k=1}^{2q} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$\prod_{k=2q+2}^{4q+1} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$\prod_{k=4q+3}^{6q+2} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$\prod_{k=6q+4}^{8q+3} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$\prod_{k=8q+5}^{10q+4} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$= (-2)(-2)(-2)(-2)(8)(8^{10q}) \prod_{k=1}^{2q} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$\prod_{k=2q+2}^{4q+1} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$\prod_{k=4q+3}^{6q+2} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$\prod_{k=6q+4}^{8q+3} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5})$$

$$\prod_{k=8q+5}^{10q+4} (\cos \frac{k\pi}{10q+5} \cos \frac{2k\pi}{10q+5} \cos \frac{5k\pi}{10q+5}).$$

Using Lemma 2.2, we have

$$det(A(C_n^4)) = (-2)(-2)(-2)(-2)(8)(8^{10q})(8^{-10q})$$
$$= 128$$

Therefore $det(A(C_n^4)) = 128$ when $n \equiv 5 \bmod 10$. From case I, II and III, then

$$det(A(C_n^4)) = \begin{cases} 0 & ; n \equiv 0,2,4,6,8 \bmod 10 \\ 8 & ; n \equiv 1,3,7,9 \bmod 10 \\ 128 & ; n \equiv 5 \bmod 10. \end{cases}$$

<div align="right">□</div>

Moreover, observe that $det(A(C_n^d)) = 0$ where $n = 2 + 2d$ and d are even, we have Corollary 2.4.

Corollary 2.4. *If $n = 2 + 2d$ and d is even then the adjacency matrix of cycle-power graph C_n^d is singular .*

Proof. Let d be even. By proposition 1.1, we have

$$E(C_n^d; k) = z + z^2 + \ldots + z^d + z^{n-d} + \ldots + z^{n-2} + z^{n-1}$$
$$= 2\cos\frac{2k\pi}{n} + 2\cos\frac{4k\pi}{n} + \ldots + 2\cos\frac{2kd\pi}{n}$$
$$= 2\sum_{j=1}^{d}\cos\frac{2jk\pi}{n} \tag{5}$$

Since $n = 2 + 2d$ and $1 \le k \le n$. Consider (5) when $k = \frac{n}{2}$. Then

$$E(C_n^d; \frac{n}{2}) = 2\cos\frac{2(\frac{n}{2})\pi}{n} + 2\cos\frac{4(\frac{n}{2})\pi}{n} + \ldots + 2\cos\frac{2(\frac{n}{2})d\pi}{n}$$
$$= 0.$$

Therefore the adjacency matrix of C_n^d is singular matrix. □

References

[1] A. Abdollahi, *Determinant of adjacency matrices of graph*, http://arxiv.org/abs/0908.3324, Cornell University.

[2] N. Adsawatithisakul and D. Samana, Determinant of adjacency matrix of square cycle graph, International Journal of Pure and Applied Mathematics, 2014.

[3] N. Biggs,Algebraic Graph Theory, Cambridge University Press, Cambridge, 1974.

[4] C.N. Campos and C.P. De Mello, A result on the total colouring of powers of cycles, Electronic Notes in Discrete Mathematics, 18, 2004, 47-52.

[5] D.M. Cvetkovic, M. Doob and H. Sachs, Spectra of Graphs: Theory and Application, Academic Press, New York, 1980.

[6] M. Doob, Circulant graphs with $det(-A(G)) = -deg(G)$:codeterminants with K_n,Linear Algebra Appl, 340, 2002, 87-96.

[7] L. Goldberg, Matrix Theory with Applications, McGraw - Hill International Editions, Mathematics and Statistics Series, 1991.

[8] B. Gyurov and J. Cloud, *On the algebraic properties of Pin-Wheel graphs and Applications*, 73rd annual meeting of the Oklahoma Arkansas section 2011, University of central Oklahoma.

[9] Y. Hoa, C. Woo and P. Chen, On the sandpile group of the square cycle C_n^2, Linear Algebra Appl, 418, 2006, 457-467.

[10] S. Hu, The Classification and maximum determinants of the adjacency matrices of graphs with one cycle, J. Math. Study, 36, 2003, 102-104.

[11] M. Krivelevich and A. Nachmias, Colouring powers of cycles from random lists, European Journal of Combinatorics, 25, 2004, 961-968.

[12] D.Li and M.Liu,Hadwiger's conjecture for powers of cycles and their complements,European Journal of Combinatorics, 28, 2007, 1152-1155.