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TRIPOLAR FUZZY INTERIOR IDEALS OF SEMIRINGS

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Abstract

In this paper, we introduce the notion of tripolar fuzzy set to be able to deal with tripolar information as a generalization of fuzzy set, bipolar fuzzy set and intuitionistic fuzzy set. The tripolar fuzzy set representation is very useful in discriminating relevant elements, irrelevant elements and contrary elements.We also introduce the notion of tripolar fuzzy ideal and tripolar fuzzy interior ideal of semiring. We study some of their algebraic properties, relations between them and characterization of tripolar fuzzy interior ideals are given.

1 Introduction

Historically semirings first appear implicitly in Dedekind and later in Macaulay, Noether and Lorenzen in connection with the study of rings. However semirings first appear explicitly in Vandiver [8], also in connection with the axiomatization of Arithmetic of natural numbers. Semirings have been studied by various

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researchers in an attempt to broaden techniques coming from semigroup theory, ring theory or in connection with applications. The developments of the theory in semirings have been taking place since 1950. Semirings abound in the Mathematical world around us. A semiring is one of the fundamental structures in Mathematics. Indeed the first Mathematical structure we encounter the set of natural numbers is a semiring. Other semirings arise naturally in such diverse areas of Mathematics as combinatorics, functional analysis, topology, graph theory, Euclidean geometry, probability theory, commutative ring theory, the Mathematical modeling of quantum physics and parallel computation system.

The theory of fuzzy sets most appropriate theory for dealing with uncertainty was first introduced by Zadeh [9] in 1965. There are many extensions of fuzzy sets, for example, intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, bipolar fuzzy sets, cubic sets etc. The concept of fuzzy set was applied to theory of subgroups by Rosenfeld [7]. The notion of an intuitionistic fuzzy set was first introduced by Atanassov [1] as a generalization of notion of fuzzy set.

Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is $[-1, 1]$. In 1994, Zhang [10] initiated the concept bipolar fuzzy sets as a generalization of fuzzy sets. In 2000, Lee [4, 2] and Lee [5, 6] used the term bipolar valued fuzzy sets and applied it to algebraic structure. Kim et al. [3] studied intuitionistic fuzzy interior ideals in semigroups, introduced the notion of bipolar fuzzy ideals and bipolar fuzzy filters in CI-algebras.

In this paper, we introduce the notion of tripolar fuzzy set to be able to deal with tripolar information as a generalization of fuzzy set, bipolar fuzzy set and intuitionistic fuzzy set. The tripolar fuzzy set representation is very useful in discriminating relevant elements, irrelevant element s and contrary elements. We introduce the notion of tripolar fuzzy ideals and tripolar fuzzy interior ideals of semiring and study some of their algebraic properties and relations between them. Characterization of tripolar fuzzy interior ideals are given. We also prove that for any homomorphism ϕ from a semiring M to a semiring N, if A is a tripolar fuzzy interior ideal of M then the homomorphic image $\phi(A)$ is a tripolar fuzzy interior ideal of N and B is a tripolar fuzzy interior ideal of N then the homomorphic pre image $\phi^{-1}(B)$ is a tripolar fuzzy interior ideal of M.

2 Preliminaries

In this section, we recall some definitions introduced in this field earlier.

Definition 2.1. *A universal algebra $(S, +, \cdot)$ is called a semiring if and only if $(S, +), (S, \cdot)$ are semigroups which are connected by distributive laws, i.e., $a(b + c) = ab + ac, \quad (a + b)c = ac + bc,$ for all $a, b, c \in S$.*

A semiring M is said to be commutative semiring if $xy = yx$, for all $x, y \in M$. A semiring M is said to have zero element if there exists an element $0 \in M$

such that $0 + x = x = x + 0$ and $0 \cdot x = x \cdot 0 = 0$, for all $x \in M$. An element $1 \in M$ is said to be unity if for each $x \in M$ such that $x \cdot 1 = 1 \cdot x = x$. In a semiring M with unity 1, an element $a \in M$ is said to be left invertible (right invertible) if there exists $b \in M$ such that $ba = 1(ab = 1)$. In a semiring M with unity 1, an element $a \in M$ is said to be invertible if there exists $b \in M$ such that $ab = ba = 1$. A semiring M with unity 1 is said to be division semiring if every non zero element of M is invertible. An element $a \in M$ is said to be regular element of M if there exists $x \in M$ such that $a = axa$. If every element of semiring M is a regular then M is said to be regular semiring. An element $a \in M$ is said to be idempotent of M if $a = a^2$. Every element of M is an idempotent of M then M is said to be idempotent semiring M.

Definition 2.2. *A non-empty subset A of semiring M is called*

(i) a subsemiring of M if $(A, +)$ is a subsemigroup of $(M, +)$ and $AA \subseteq A$.

(ii) a quasi ideal of M if A is a subsemiring of M and $AM \cap MA \subseteq A$.

(iii) a bi-ideal of M if A is a subsemiring of M and $AMA \subseteq A$.

(iv) an interior ideal of M if A is a subsemiring of M and $MAM \subseteq A$.

(v) a left (right) ideal of M if A is a subsemiring of M and $MA \subseteq A(AM \subseteq A)$.

(vi) an ideal if A is a subsemiring of M, $AM \subseteq A$ and $MA \subseteq A$.

(vii) a $k-$ideal if A is a subsemiring of M, $AM \subseteq A, M\ A \subseteq A$ and $x \in M$, $x + y \in A, y \in A$ then $x \in A$.

(viii) a left(right) bi-quasi ideal of M if A is a subsemiring of M and $MA \cap MAM(AM \cap MAM) \subseteq A$.

(ix) a bi-quasi ideal of M if A is a left bi- quasi ideal and a right bi- quasi ideal of M

A semiring M is a left (right) simple semiring if M has no proper left (right) ideal of M. A semiring M is a bi-quasi simple semiring if M has no proper bi-quasi ideal of M. A semiring M is said to be simple semring if M has no proper ideals.

Let S be a non-empty set. Then a mapping $f : S \to [0, 1]$ is called a fuzzy subset of M. A fuzzy subset $\mu : S \to [0, 1]$ is non-empty if μ is not the constant function. For any two fuzzy subsets λ and μ of S, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in S$. The complement of a fuzzy subset μ of a semiring S is denoted by $\overline{\mu}$ and it is defined as $\overline{\mu}(x) = 1 - \mu(x)$, for all $x \in S$.

Definition 2.3. *A bipolar fuzzy set A of a non-empty set X is an object having the form $A = \{(x, \mu_A(x), \delta_A(x)) \mid x \in X\}$, Where $\mu_A : X \to [0, 1]$; $\delta_A : X \to$*

$[-1, 0]$. $\mu_A(x)$ represents degree of satisfaction of an element x to the property corresponding to fuzzy set A and $\delta_A(x)$ represents degree of satisfaction of an element x to the implicit counter property of fuzzy set A.

Definition 2.4. *A fuzzy subset μ of semiring S is called a fuzzy interior ideal if*

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(xyz) \geq \mu(y)$, *for all* $x, y, z \in S$.

Definition 2.5. *Let $\phi : S \to S^1$ be a homomorphism of semirings. Let f be a fuzzy subset of S. We define a fuzzy subset $\phi(f)$ of S^1 by*

$$\phi(f)(x) = \begin{cases} \sup\limits_{y \in \phi^{-1}(x)} f(y), & \text{if } \phi^{-1}(x) \neq \emptyset, \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.6. *Let $\phi : S \to S^1$ be a homomorphism of semirings S, S^1 and μ be a fuzzy subset of S. Then μ is said to be ϕ homomorphism invariant if $\phi(a) = \phi(b)$ then $\mu(a) = \mu(b)$, for all $a, b \in S$.*

Theorem 2.1. *Let S and S^1 be semirings, $\phi : S \to S^1$ be a homomorphism and f be a ϕ invariant fuzzy ideal of semiring S. If $x = \phi(a)$ then $\phi(f)(x) = f(a), a \in S$.*

3 TRIPOLAR FUZZY INTERIOR IDEALS OF SEMIRING

In this section, we introduce the notion of tripolar fuzzy set to be able to deal with tripoalr information as a generalization of fuzzy set, bipolar fuzzy set and intuitionistic fuzzy set. We also introduce the notion of tripolar fuzzy ideals and interior ideals of semiring.

Definition 3.1. *A tripolar fuzzy set A in a universe set X is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x), \delta_A(x)) \mid x \in X$ and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1\}$. Where $\mu_A : X \to [0, 1]; \lambda_A : X \to [0, 1]; \delta_A : X \to [-1, 0]$ such that $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$. The membership degree $\mu_A(x)$ characterizes the extent that the element x satisfies to the property corresponding to tripolar fuzzy set A, $\lambda_A(x)$ characterizes the extent that the element x satisfies to the not property (irrelevant) corresponding to tripolar fuzzy set A and $\delta_A(x)$ characterizes the extent that the element x satisfies to the implicit counter property of tripolar fuzzy set A. For simplicity $A = (\mu_A, \lambda_A, \delta_A)$ has been used for $A = \{(x, \mu_A(x), \lambda_A(x), \delta_A(x)) \mid x \in X, 0 \leq \mu_A(x) + \lambda_A(x) \leq 1\}$.*

Remark 3.1. *A tripolar fuzzy set A is a generalization of a bipolar fuzzy set and an intuitionistic fuzzy set. A tripolar fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x), \delta_A(x)) \mid x \in X\}$ represents the sweet taste of food stuffs. Assuming the sweet taste of* *food stuff as a positive membership value $\mu_A(x)$ i.e., the element x is satisfying the sweet property. Then bitter taste of food stuff as a negative membership value $\delta_A(x)$ i.e., the element x is satisfying the bitter property, and the remaining tastes of food stuffs like acidic, chilly etc., as a non memberships value $\lambda_A(x)$ i.e., the element is satisfying irrelevant to the sweet property.*

Definition 3.2. *A tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ of a semiring M is called a tripolar fuzzy subsemiring of M if A satisfies the following conditions*

(i) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(x)\}$

(ii) $\lambda_A(x + y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$

(iii) $\delta_A(x + y) \leq \max\{\delta_A(x), \delta_A(y)\}$

(iv) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(x)\}$

(v) $\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\}$

(vi) $\delta_A(xy) \leq \max\{\delta_A(x), \delta_A(y)\}$, *for all* $x, y \in M$.

Definition 3.3. *A tripolar fuzzy subsemiring $A = (\mu_A, \lambda_A, \delta_A)$ of a semiring M is called a tripolar fuzzy ideal of M if A satisfies the following conditions.*

(i) $\mu_A(xy) \geq \max\{\mu_A(x), \mu_{A)}(x)\}$

(ii) $\lambda_A(xy) \leq \min\{\lambda_A(x), \lambda_A(y)\}$

(iii) $\delta_A(xy) \leq \min\{\delta_A(x), \delta_A(y)\}$, *for all* $x, y \in M$.

Definition 3.4. *A tripolar fuzzy subsemiring $A = (\mu_A, \lambda_A, \delta_A)$ of a semiring M is called a tripolar fuzzy interior ideal of M if A satisfies the following conditions.*

(i) $\mu_A(xzy) \geq \mu_A(z)$

(ii) $\lambda_A(xzy) \leq \lambda_A(z)$

(iii) $\delta_A(xzy) \leq \delta_A(z)$, *for all* $x, y, z \in M$.

Theorem 3.2. *Every tripolar fuzzy ideal of a semiring M is a tripolar fuzzy interior ideal of a semiring M.*

Proof. Let $A = (\mu_A, \lambda_A, \delta_A)$ be a tripolar fuzzy ideal of a semiring M. Then

(i) $\mu_A(xzy) \geq \mu_A(xz) \geq \mu_A(z)$

(ii) $\lambda_A(xzy) \leq \lambda_A(xz) \leq \lambda_A(z)$

(iii) $\delta_A(xzy) \leq \delta_A(xz) \leq \delta_A(z)$, for all $x, y, z \in M$.

Hence A is a tripolar fuzzy interior ideal of M. □

Remark 3.3. *Every tripolar fuzzy ideal A of a semiring M is a tripolar fuzzy subsemiring M but the converse is not true.*

Example 3.1. *Let $M = \{x_1, x_2, x_3\}$. We define operations with the following tables*

$+$	x_1	x_2	x_3
x_1	x_1	x_2	x_3
x_2	x_2	x_2	x_3
x_3	x_3	x_3	x_2

;

$\cdot$	x_1	x_2	x_3
x_1	x_1	x_3	x_3
x_2	x_3	x_2	x_3
x_3	x_3	x_3	x_3

Then M is a semiring. B is a tripolar fuzzy set defined as
$B = \{(x_1, 0.2, 0.7, -0.2), (x_2, 0.3, 0.6, -0.3), (x_3, 0.6, 0.3, -0.3)\}$
Then B is a tripolar fuzzy subsemiring of M.
Here B is a tripolar fuzzy interior ideal of M but not a tripolar fuzzy ideal of M.

Theorem 3.4. *Every tripolar fuzzy interior ideal over a regular semiring M is a tripolar fuzzy ideal of M.*

Proof. Let $A = (\mu_A, \lambda_A, \delta_A)$ be a tripolar fuzzy interior ideal of a regular semiring M. Suppose $x, y \in M$. Then there exists $z \in M$ such that $xy = xyzxy$.

$$\begin{aligned} \mu_A(xy) &= \mu_A(xyzxy) = \mu_A\big(xy(zxy)\big) \geq \mu_A(y) \\ \mu_A(xy) &= \mu_A\big((xyz)xy\big) \geq \mu_A(x). \end{aligned}$$

Therefore μ_A is a fuzzy ideal of M.

$$\begin{aligned} \lambda_A(xy) &= \lambda_A(xyzxy) \leq \lambda_A(y) \\ \lambda_A(xy) &= \lambda_{A)}\big((xyz)xy\big) \leq \lambda_A(x). \end{aligned}$$

Therefore λ_A is a fuzzy ideal of M.

$$\begin{aligned} \delta_A(xy) &= \delta_A(xyzxy) \leq \delta_A(y) \\ \delta_A(xy) &\leq \delta_A(x). \end{aligned}$$

Therefore δ_A is a fuzzy ideal of M.
Hence A is a tripolar fuzzy ideal of semiring M. □

Theorem 3.5. *If a tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ of semiring M is an interior ideal of semiring M then $(\mu_A, \overline{\mu}_A, \delta_A)$ is a tripolar fuzzy interior ideal of semiring M.*

Proof. Let $x, y \in M$.

$$\begin{aligned} \overline{\mu}_A(xy) &= 1 - \mu_A(xy) \leq 1 - \min\{\mu_A(x), \mu_A(y)\} = \max\{1 - \mu_A(x), 1 - \mu_A(y)\} \\ &= \max\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}. \end{aligned}$$

$$\overline{\mu}_A(xzy) = 1 - \mu_A(xzy) \leq 1 - \mu_A(z) = \overline{\mu}_A(z).$$

Therefore $(\mu_A, \overline{\mu}_A, \delta_A)$ is a tripolar fuzzy interior ideal of semiring M. □

Definition 3.5. *Let $A = (\mu_A, \lambda_A, \delta_A)$ be a tripolar fuzzy set of semiring M and $t \in [0,1]$. Then the sets $\mu_{A,t} = \{x \in M \mid \mu_A(x) \geq t\}; \lambda_{A,t} = \{x \in M \mid \lambda_A(x) \leq t\}; \delta_{A,t} = \{x \in M \mid \delta_A(x) \leq -t\}$ are called a $\mu-$level $t-$cut, $\lambda-$level $t-$cut and $\delta-$level $-t-$cut of A respectively.*

Theorem 3.6. *If $A = (\mu_A, \lambda_A, \delta_A)$ is a tripolar fuzzy interior ideal of semiring M then $\mu-$level $t-$cut, $\lambda-$level $t-$cut and $\delta-$level $-t-$cut of A are interior ideals of semiring M, for all $t \in Im(\mu_A) \cap Im(\lambda_A) \subseteq [0,1]$ and $-t \in Im(\delta_A)$.*

Proof. Let $t \in Im(\mu_A) \cap Im(\lambda_A) \subseteq [0,1]$ and $t \in Im(\delta_A)$ and $x, y \in \mu_{A,t}$. Then $\mu_A(x) \geq t$ and $\mu_A(y) \geq t$.

$$\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq t \;\Rightarrow\; x+y \in \mu_{A,t}.$$
$$\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \geq t \;\Rightarrow\; xy \in \mu_{A,t}.$$

Hence $\mu_{A,t}$ is a subsemiring of M.

Let $x, y \in M, z \in \mu_{A,t}$. Then $\mu_A(xzy) \geq \mu_A(z) \geq t \;\Rightarrow\; xzy \in \mu_{A,t}$.
Hence $\mu_{A,t}$ is an interior ideal of semiring M.
Suppose $x, y \in \lambda_{A,t}$. Then $\lambda_A(x) \leq t, \lambda_A(y) \leq t$

$$\Rightarrow \lambda_A(x+y) \leq \max\{\lambda_A(x), \lambda_A(y)\} \leq t.$$
$$\text{Therefore } x+y \in \lambda_{A,t}.$$
$$\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\} \leq t.$$
$$\text{Therefore } xy \in \lambda_{A,t}.$$

Hence $\lambda_{A,t}$ is a subsemiring of M.

Let $x, y \in M, z \in \lambda_{A,t}$. Then $\lambda_A(xzy) \leq \lambda_A(z) \leq t \;\Rightarrow\; xzy \in \lambda_{A,t}$.

Suppose $x, y \in \delta_{A,-t}$. Then $\delta_A(x) \leq -t, \delta_A(y) \leq -t$.

$$\delta_A(x+y) \leq \max\{\delta_A(x), \delta_A(y)\} \leq -t$$
$$\text{Therefore } x+y \in \delta_{A,-t}.$$
$$\delta_A(xy) \leq \max\{\delta_A(x), \delta_A(y)\} \leq -t$$
$$\text{Therefore } xy \in \delta_{A,-t}.$$

Let $x, y \in M, z \in \delta_{A,-t}$. Then $\delta_A(xzy) \leq \delta_A(z) \leq -t$.
Therefore $xzy \in \delta_{A,-t}$. Hence $\delta_{A,-t}$ is an interior ideal of semiring M. □

The following proof of the theorem is similar to proof of the Theorem [3.9] in [3]. Hence we omit the proof of the following theorem.

Theorem 3.7. *Let $A = (\mu_A, \lambda_A, \delta_A)$ be a tripolar fuzzy set in semiring M such that non-empty sets $\mu_{A,t}, \lambda_{A,t}, \delta_{A,t}$ are interior ideals of M for all $t \in [0,1]$. Then A is a tripolar fuzzy interior ideal of M.*

Theorem 3.8. *A tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ is a fuzzy interior ideal of semiring M if and only if fuzzy subsets $\mu_A, \overline{\lambda}_A, \delta_A$ are fuzzy interior ideals of semiring M.*

Proof. Suppose $A = (\mu_A, \lambda_A, \delta_A)$ is a tripolar fuzzy interior ideal of semiring M. Then obviously μ_A, δ_A are fuzzy interior ideals of M. Let $x, y \in M$.

$$\begin{aligned}
\overline{\lambda}_A(x+y) &= 1 - \lambda_A(x+y) \\
&\geq 1 - \max\{\lambda_A(x), \lambda_A(y)\} \\
&= \min\{1 - \lambda_A(x), 1 - \lambda_A(y)\} \\
&= \min\{\overline{\lambda}_A(x), \overline{\lambda}_A(y)\}. \\
\overline{\lambda}_A(xy) &= 1 - \lambda_A(xy) \\
&\geq 1 - \max\{\lambda_A(x), \lambda_A(y)\} \\
&= \min\{1 - \lambda_A(x), 1 - \lambda_A(y)\} \\
&= \min\{\overline{\lambda}_A(x), \overline{\lambda}_A(y)\}, \text{ for all } x, y \in M.
\end{aligned}$$

Suppose $x, y, z \in M,$. Then $\overline{\lambda}_A(xzy) = 1 - \lambda_A(xzy) \geq 1 - \lambda_A(z) = \overline{\lambda}_A(z)$. Hence $\overline{\lambda}$ is an fuzzy interior ideal of M.

Conversely suppose that $\mu_A, \overline{\lambda}_A, \delta_A$ are fuzzy interior ideals of semring M. Let $x, y, z \in M$.

$$\begin{aligned}
\lambda_A(x+y) = 1 - \overline{\lambda}_A(x+y) \geq \min\{1 - \overline{\lambda}_A(x), 1 - \overline{\lambda}_A(y)\} = \min\{\lambda_A(x), \lambda_A(y)\}. \\
\lambda_A(xy) = 1 - \overline{\lambda}_A(xy) \geq \max\{1 - \overline{\lambda}_A(x), 1 - \overline{\lambda}_A(y)\} = \max\{\lambda_A(x), \lambda_A(y)\}. \\
\overline{\lambda}_A(xzy) \geq \overline{\lambda}_A(z) \Rightarrow 1 - \lambda_A(xzy) \geq 1 - \lambda_A(z) \Rightarrow \lambda_A(xzy) \leq \lambda_A(z).
\end{aligned}$$

Hence the theorem. □

Corollary 3.9. *A tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ is a tripolar fuzzy interior ideal of semiring M if and only if the fuzzy sets $(\mu_A, \overline{\mu}_A, \delta_A)$ and $(\overline{\lambda}_A, \lambda_A, \delta_A)$ are tripolar fuzzy interior ideals of semiring M.*

Definition 3.6. *Let $f : X \to Y$ be a map. If $B = (\mu_B, \lambda_B, \delta_B)$ is a tripolar fuzzy set in Y. Then pre-image of B under f, denoted by $f^{-1}(B)$, is a tripolar fuzzy set in X defined by*

$$f^{-1} = \big(f^{-1}(\mu_B), f^{-1}(\lambda_B), f^{-1}(\delta_B)\big),$$

where $f^{-1}(\mu_B) = \mu_B(f), f^{-1}(\lambda_B) = \lambda_B(f)$ and $f^{-1}(\delta_B) = \delta_B(f)$.

Theorem 3.10. *Let $f : M \to N$ be a homomorphism of semrings. If $B = (\mu_B, \lambda_B, \delta_B)$ is a tripolar fuzzy interior ideal of semiring N. Then $f^{-1}(B) = \big(f^{-1}(\mu_B), f^{-1}(\lambda_B), f^{-1}(\delta_B)\big)$ is a tripolar fuzzy interior ideal of semiring M.*

Proof. Suppose $B = (\mu_B, \lambda_B, \delta_B)$ is a tripolar fuzzy interior ideal of semiring N and $x, y \in M$. Then

$$\begin{aligned}
f^{-1}\big(\mu_B(x+y)\big) &= \mu_B(f(x+y)) = \mu_B\big(f(x)+f(y)\big) \\
&\geq \min\{\mu_B(f(x)), \mu_B(f(y))\} \\
&= \min\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(y))\} \\
f^{-1}\big(\mu_B(xy)\big) &= \mu_B(f(xy)) = \mu_B\big(f(x), f(y)\big) \geq \min\{\mu_B(f(x)), \mu_B(f(y))\} \\
&= \min\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(y))\}.
\end{aligned}$$

Suppose that $x, y, z \in M$. Then we have

$$\begin{aligned}
f^{-1}\big(\mu_B(xzy)\big) &= \mu_B(f(xzy)) = \mu_B\big(f(x)f(z)f(y)\big) \geq \mu_B(f(z)) \\
&= f^{-1}(\mu_B f(z)). \\
f^{-1}\big(\lambda_B(x+y)\big) &= \lambda_B(f(x+y)) = \lambda_B\big(f(x)+f(y)\big) \\
&\leq \max\{\lambda_B(f(x)), \lambda_B(f(y))\} \\
&= \max\{f^{-1}(\lambda_B(x)), f^{-1}(\lambda_B(y))\}. \\
f^{-1}\big(\lambda_B(xy)\big) &= \lambda_B(f(xy)) = \lambda_B\big(f(x)f(y)\big) \\
&\leq \max\{\lambda_B(f(x)), \lambda_B(f(y))\} \\
&= \max\{f^{-1}(\lambda_B(x)), f^{-1}(\lambda_B(y))\}. \\
f^{-1}\big(\lambda_B(xzy)\big) &= \lambda_B(f(xzy)) = \lambda_B\big(f(x)f(z)f(y)\big) \leq \lambda_B(f(z)) \\
&= f^{-1}(\lambda_B(f(z))). \\
f^{-1}\big(\delta_B(x+y)\big) &= \delta_B(f(x+y)) = \delta_B\big(f(x)+f(y)\big) \leq \max\{\delta_B(f(x)), \delta_B(f(y))\} \\
&= \max\{f^{-1}(\delta_B(x)), f^{-1}(\delta_B(y))\}. \\
f^{-1}\big(\delta_B(xy)\big) &= \delta_B(f(xy)) = \delta_B\big(f(x)f(y)\big) \leq \max\{\delta_B(f(x)), \delta_B(f(y))\} \\
&= \max\{f^{-1}(\delta_B(x)), f^{-1}(\delta_B(y))\}. \\
f^{-1}\big(\delta_B(xzy)\big) &= \delta_B(f(xzy)) = \delta_B\big(f(x)f(z)f(y)\big) \leq \delta_B(f(z)) \\
&= f^{-1}(\delta_B f(z)).
\end{aligned}$$

Hence $f^{-1}(B) = \big(f^{-1}(\mu_B), f^{-1}(\lambda_B), f^{-1}(\delta_B)\big)$ is a tripolar fuzzy interior ideal of semiring M. $\square$

Theorem 3.11. *Let M and N be semirings, $\phi : M \to N$ be a homomorphism and f be a ϕ invariant fuzzy ideal of semiring M. If $x = \phi(a)$ then $\phi(f)(x) = f(a), a \in M$.*

Proof. Let M and N be semirings, $a \in M, x \in N$, $x = \phi(a)$.
Then $a \in \phi^{-1}(x)$ and $t \in \phi^{-1}(x)$.
Therefore $\phi(t) = x = \phi(a)$, since f is ϕ invariant.

$f(t) = f(a) \Rightarrow \phi(f)(x) = \sup_{t \in \phi^{-1}(x)} \{f(t)\} = f(a).$

Hence $\phi(f)(x) = f(a)$. □

Definition 3.7. *Let $\phi : M \to N$ be a homomorphism of semirings M, N and $A = (\mu_A, \lambda_A, \delta_A)$ be a tripolar fuzzy set of semiring M. Then A is said to be ϕ homomorphism invariant if $\phi(a) = \phi(b)$ then*

(i) $\mu_A(x) = \mu_A(y)$

(ii) $\lambda_A(x) = \lambda_A(y)$

(iii) $\delta_A(x) = \delta_A(y)$, *for all $x, y \in M$.*

Theorem 3.12. *Let M and N be semirings and $\phi : M \to N$ be an onto homomorphism. If A is a homomorphism ϕ invariant tripolar interior ideal of semiring M then image of A under homomorphism ϕ is a tripolar fuzzy interior ideal of semiring M.*

Proof. Let $A = (\mu_A, \lambda_A, \delta_A)$ be a tripolar interior ideal of semiring M and $x, y \in N$. Then there exist $a, b \in M$ such that $\phi(a) = x, \phi(b) = y$

$$\begin{aligned} \Rightarrow \phi(a+b) &= x + y \\ \Rightarrow \phi(\mu_A)(x+y) &= \mu_A(a+b) \\ &\geq \min\{\mu_A(a), \mu_A(b)\} \\ &= \min\{\phi(\mu_A)(x), \phi(\mu_A)(y)\}. \\ \phi(\mu_A(xy)) &= \mu_A(ab) \\ &\geq \min\{\mu_A(x), \mu_A(b)\} \\ &= \min\{\phi(\mu_A)(x), \phi(\mu_A)(y)\}. \end{aligned}$$

Suppose $x, y, z \in N$. Then there exist $a, b, c \in M$ such that $\phi(a) = x, \phi(b) = y$ and $\phi(c) = z$. Then

$$\phi(\mu_A(xzy)) = \mu_A(acb) \geq \mu_A(c) = \phi(\mu_A(z)).$$

Hence $\phi(\mu_A)$ is a fuzzy interior ideal of semiring M.

$$\begin{aligned} \phi(\lambda_A)(x+y) &= \lambda_A(a+b) \leq \max\{\lambda_A(a), \lambda_A(b)\} = \max\{\phi(\lambda_A)(x), \phi(\lambda_A)(y)\}. \\ \phi(\lambda_A(xy)) &= \lambda_A(ab) \leq \min\{\lambda_A(x), \lambda_A(b)\} = \min\{\phi(\lambda_A)(x), \phi(\lambda_A)(y)\}. \\ \phi(\lambda_A(xzy) &= \lambda_A(acb) \leq \lambda_A(c) = \phi(\lambda_A(z)). \end{aligned}$$

Therefore $\phi(\lambda_A)$ is a fuzzy interior ideal of semiring M.

$$\begin{aligned} \phi(\delta_A)(x+y) &= \delta_A(a+b) \leq \max\{\delta_A(a), \delta_A(b)\} = \max\{\phi(\delta_A)(x), \phi(\delta_A)(y)\}. \\ \phi(\delta_A(xy)) &= \delta_A(ab) \leq \min\{\delta_A(x), \delta_A(b)\} = \min\{\phi(\delta_A)(x), \phi(\delta_A)(y)\}. \\ \phi(\delta_A(xzy) &= \delta_A(acb) \leq \delta_A(c) = \phi(\delta_A(z)). \end{aligned}$$

Therefore $\phi(\delta_A)$ is a fuzzy interior ideal of semiring M. Hence $\phi(A)$ is a tripolar fuzzy interior ideal of semiring M. □

4 Conclusion

In this paper, we introduced the notion of tripolar fuzzy set to be able to deal with tripoalr information as a generalization of fuzzy set, bipolar fuzzy set and intuitionistic fuzzy set. We also introduced the notion of tripolar fuzzy ideals and tripolar fuzzy interior ideals of semiring. We studied some of their algebraic properties and relations between them.

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