***Southeast-Asian J. of Sciences*** Vol. 2, No. 2(2013) pp. 204-212

# MODELING OF PERIODIC PHENOMENA IN SECONDARY EDUCATION

**Nguyen Thi Nga**

*Department of Mathematics,*
*Ho Chi Minh city University of Pedagogy*
*Ho Chi Minh city, Vietnam*
*e-mail: ngathi103@yahoo.com*

**Abstract**

This presentation covers a comparative synthesis of secondary educational institutions of mathematics and physics in Vietnam and France on the concept of periodicity. Then, the analysis of results of a survey in Vietnam with a questionnaire on modeling of periodic phenomena allows us to clarify the concept of periodicity related conditions of his teaching.

## 1 Introduction

The periodicity is a concept used in physics and many other scientific disciplines because it is central to the study of cyclical phenomena and oscillatory phenomena. We find this concept in mathematics through the notion of periodic function. Periodic functions including trigonometric functions were formed gradually in science as a tool for modeling variable quantities that regularly return to the same state indefinitely.

Cyclic or oscillatory phenomena are present very early in the teaching of physics, chemistry and biology. When, in secondary education, it uses a mathematical formulation to support the study of a periodic phenomenon. This formalization is conducted, through mathematical models, leading to educational phenomena that our present work tries to make clarify both in Vietnam and in France.

**Key words:** periodic phenomena, periodic function, modeling.

# 2 The scientific concept of periodicity: a comparative analysis of teaching in Vietnam and France

With this analysis, it is:

- describe, in characterizing the institutional relationships with the notions of periodicity and periodic function in both institutions of secondary education in France and Vietnam
- clarify the institutional conditions and constraints on modeling of periodic phenomena in secondary mathematics education in France and Vietnam

## 2.1 Teaching of physics: two models

Table 1 presents the curriculum of study of periodic phenomena in secondary physics education.

| **Class** | **In Vietnam** | **In France** |
|---|---|---|
| 9 | no periodic phenomenon taught in college | periodic voltage, sine wave: period, frequency |
| 10 | rotation of the planets in the solar system, uniform circular motion: angular velocity, acceleration, frequency | alternation of day and night, the phases of the moon, rotation, angular velocity |
| 12 | - Harmonic oscillation (simple pendulum, physical pendulum): period, frequency, amplitude, angular frequency.<br>- The sine wave<br>- Alternative current | - Periodic wave, sine wave<br>- Electrical oscillator<br>- Simple pendulum |

Table 1: Periodic phenomena studied in physics

Both institutions show the objective of studying variable quantities with time: voltages, distances, angles, etc... In the background, even if not formally appointed, is still a periodic function whose independent variable is time.

More prevalent in Vietnam than in France, the mathematization of concepts, however, present in the two institutions and it is enriched in the curriculum based on what we can call two models: uniform circular motion (C) and the harmonic oscillation (O).

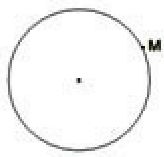
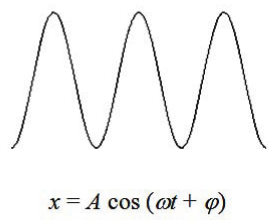
| Model C | Model O |
|---|---|
|   circular path and $\omega$ constant |   |

Table 2: Two mathematics models of periodicity

Introduced from the college in France, the harmonic oscillation is only studied by graphs taken as a result of measurement (eg waveform). The role of the graphic is clearly behind in Vietnam, where it is the algebraic register dominates. But the algebraic register finds its place after the study of trigonometric functions in mathematics in class 11. This is why the harmonic oscillation is presented in Vietnam at the end of high school.

Our epistemological analysis shows that for physicists, these two models, uniform circular motion (C) and the harmonic oscillation (O), are the basic models for the study of temporal periodic phenomena.

- Model C, characterized by a circular path and a constant angular velocity, can appear in two registers: algebraic ($x = R\cos\theta, y = R\sin\theta, \theta = \omega t$) and graphic (circle)

- Model O, present in two registers - algebraic $x = A\cos(\omega t + \varphi)$ or $x'' + \omega^2 x = 0$ and graphic (sine wave) - is a functional model that the central mathematical object is a trigonometric function.

The model C is confined to the mechanics as the basic model for the study of cyclical phenomena reducible to the movement of a mobile in a trajectory. It makes work in particular the concepts of velocity and acceleration.

The model O is dominant in Physics vibrations, oscillations and waves. This model is at the origin of many important mathematical developments, including Fourier analysis, which gives a central place to trigonometric functions.

The combination of these two models is presented in "Atlas of Physics", as follows:

> "The harmonic oscillation can be represented as the projection on a plane of uniform circular motion".

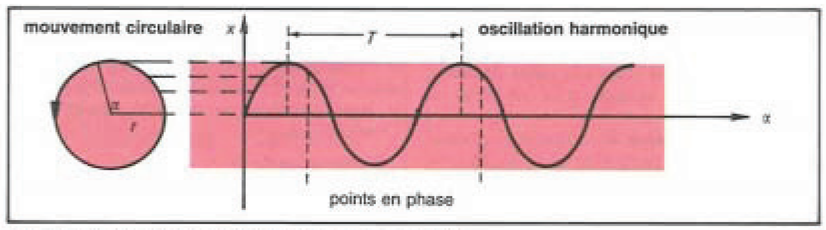



Figure 1: (Breuer (1987), p. 38-39)

For Feynman (1979), both two models are very close mathematically. The transition from one to another can simplify solving some problems.

> "We can see that uniform circular motion and vertical oscillatory motion are very similar mathematically speaking, and so we can study the oscillatory motion of a simple way by imagining as the projection of something that moves on a circle.
> [...]
> We will be able to study our oscillator in one dimension through the circular movement, which is much easier than solving a differential equation". (Feynman (1979), p. 283)

## 2.2 Mathematics education: trigonometric functions

Table 3 presents the curriculum of mathematical objects attached to the concept of periodicity in secondary mathematics education:

Beyond the common framework for the two institutions, it is important to highlight two key differences:

- The first is that in France the periodicity of a function is regarded as the property of a function as well as the parity - based on a general definition - and permit to the restriction of the interval function in task types study a numerical function. In Vietnam, conversely, the periodicity is set as a property of trigonometric functions and therefore lives only with these functions.

- The second is the association in the types of tasks that are found in France the two registers of the periodic function - like any other function: the graphic register through the graph in a Cartesian coordinate system and algebraic register through the formula explaining the relationship between the dependent variable and the independent variable. In Vietnam,

| | **In France (programme 2005)** | **In Vietnam (programme 2006)** |
|---|---|---|
| College | - periodic decimal expansion<br>- Trigonometric functions<br>- Periodic functions (graphic)<br>- Period, frequency | periodic decimal expansion |
| High School **(class 10)** | periodic decimal expansion<br>- Trigonometric functions<br>- Periodic functions (graphic)<br>- Period, frequency | |
| High School **(class 11)** | Periodic functions | Trigonometric functions |

Table 3: Objects of periodicity in mathematics

otherwise, the algebraic register dominates and the graphic register has only a supplemental role.

## 2.3 Teaching modeling?

The importance of modeling in science is emphasized by John von Neumann (1947) as follows:

> "The sciences do not try to explain, they hardly even try to interpret; they merely make models. By a model is meant a mathematical construct which, with the addition of certain verbal interpretations, describes observed phenomena". (John von Neumann, 1947)

Under the term modeling, Legrand (2003) distinguishes four different activities:

- A "pretext" modelling: using a "real" to serve as a concrete support to the mathematical model we want to teach or run;
- A "model to follow" modelling: serving as a rule of action, for example, an engineer, the model can not be explained but when applied, it works;
- A "completed scientific model": explaining and rationalizing a "real" model is presented;
- A "act of scientific modeling": starting from a more or less well-defined real and one question at a time very precise but often too large because

> too ambitious [...] The model is built from the initial question and simultaneously mathematized enough [...].

Legrand then remarked that the third activity is most prevalent in schools and universities, which is due, he says, "it pleases the mathematic teacher because it does not require to enter a philosophical and scientific debate about what care or neglect" and " it pleases the physics teacher because it ennobles his theories by the mathematic (it seems more rigorous, less controversial, easier to teach [...]).

We agree with his analysis and conclusion: both models C and O in the French and Vietnamese secondary education do live modeling activities as a "pretext" or "completed scientific model". The teaching of mathematical modeling in particular modeling of periodic phenomena can be reduced to the teaching of the use of models. In the exercises, mathematical models (C and/or O) are provided with the statement and the reality is already modeled as a pretext for mathematical work in the designated model.

In France, there are situations "modeling" where the associated functions are given by algebraic or graphical registers. The French institution seeks to exploit the phenomenon of information from its mathematical model (algebraic formula or graph). The conversion between the two registers of a function is expected institutionally.

Conversely, only the algebraic register is present in situations of modelling in Vietnamese institution. The numerical register is absent both in Vietnam and in France. It appears only after the study of variation to draw the graph of a function.

In two secondary educational institutions, there aren't tasks dedicated to the passage of one of the models C and O to another, or neither in the teaching of mathematics nor in physics. In the Vietnamese institution, although there are exercises presented both models C and O, the questions are only on the O and algebraic register, the model C has only illustrative role. The use of model C in these exercises is neither working nor expected.

We want to prove that conclusion by the exercises in Vietnamese mathematics textbooks:

> "The number of hours of sunlight in the city A in north latitude 40 degrees during the day t is given by:
>
> $$d(t) = 3\sin\left[\frac{\pi}{182}(t-80)\right] + 12, t \in \mathbb{Z} \text{ and } 0 < t \leq 365$$
>
> a) The city A has 12 hours of sunlight on which day of the year?
>
> b) On which day of the year the city A has at least hours of sunlight?
>
> c) On which day of the year the city A has the most hours of sunlight?"

(Algebra and Analysis 11 in Vietnam)

The reference to the model O allows here to better show how the results of astronomical science come from mathematics, but not to enter into a process of modelling. What is for example given the latitude and latitude is it part of model to build?

# 3 Questionnaire

Three questions are at the origin of the decision to build a questionnaire for the students:
Q1: How, in the current conditions of teaching, the student uses no mathematical knowledge to work within one of the mathematical models C and O?
Q2: How, in the current conditions of teaching, the student refers to one of the C and O mathematical models to solve a no mathematical problem?
Q3: Under what conditions the student may enter into a process of modeling a non- mathematical problem on a periodic phenomenon?

To search for answers to these questions, we designed a questionnaire composed of four exercises. All exercises are built on breaks of instructional contract both in Vietnam and in France (eg no statement does not contain the words "periodicity" or "periodic").

The statement refers to these exercises, more or less explicitly, a numerical function whose independent variable is the time that the student must recognize and use to answer questions.

For the exercises 1, 2 and 3:

- The first is an open question: "What can you say about this phenomenon?"
- The following questions suggest the use of the periodicity through the functional register - graphic, algebraic formula or numerical table - to interpolate;
- The last question "Building a figure [beyond what is given]" promotes the use of periodicity through one of two models C or O.

Exercise 4, meanwhile, does not favour any of the two models C and O. The student must participate in the modeling process to build the corresponding mathematical model, study it and answer questions about the phenomenon.

Table 4 shows the variables in the questionnaire

# 4 Analysis of the results obtained in Vietnam

The experiment took place in Vietnam for two 45-minute sessions. The questionnaire was proposed to two hundred students in the high school in Vietnam

| Exercises | Discipline of context | Class of periodic phenomena | Registers of function | Break of contract |
|---|---|---|---|---|
| 1 | biology | oscillations | graphic | Vietnam |
| 2 | physical | circular motions | algebraic | France |
| 3 | geography | oscillations | numerical | France / Vietnam |
| 4 | physical | circular motions | natural language | France / Vietnam |

Table 4: Variables of questionnaire

(class 12) after the teaching of harmonic oscillation in physics.

This survey allows us to provide answers to clarify the concept of periodicity in relation to the conditions of his teaching.

- The periodicity is understood by students as a repetition, regularity in a circular motion or oscillation. This property is of course present in the reference models C and O, but it can also be in the operation of the linear model by the analysis of exercise 4. However, periodic properties of the phenomena are rarely tools for students to solve problems that are proposed. The students were not able to connect the periodic phenomena to mathematical models, such as C and O

- Both models C and O are collected but the dominance of the local perspective on the co-variation prevents their articulation and exploitation as we can see in the exercise 4. In addition to difficulties such using of reality information to place in the mathematical model or as put in practice the results obtained from the mathematical model or reject it, the competition between the two models requires conversions between registers and reuse in an object of mathematical models that, institutionally, does not exist. Responses to the questionnaire confirmed the consequences of institutional status: they reveal the challenges facing the Vietnamese student in modeling periodic phenomena. Mainly, it is difficult to:

  + Choose, depending on the problem to be solved, one of the two models C or O
  + Specify the selected model (data and settings)
  + Pass from one model to another

  The relationship between mathematical knowledge, from models C and O, and extra-mathematical knowledge, called from the context of the exercise, is one of the areas of work to understand the learning processes at work in an activity of modeling.

- As might be expected, the entry into a modeling process is difficult for most students. Although students can generally determine a set of dependent variables, they find it difficult to construct a mathematical model. In particular, the validation step seems to be absent in the responses of most students, once the work done within the model, they are unable to assess whether the results are compatible or not the extra mathematical reality studied.

# References

[1] A. Bessot, C. Comiti, *Apport des études comparatives aux recherches en didactique des mathématiques : le cas Viêt Nam / France*, in L. Coulange & C. Hache (édits), Actes du séminaire national de didactique des mathématiques, 2008, p.171-194.

[2] H. Breuer (1987), *Dtv-Atlas zur Physik*, Version franaise de M. Meslé-Gribenski, P. Morin, M. Sénéchal-Couvercelle, Edition de Librairie Générale Française.

[3] P. Feynman (1963), *The Feyman lectures on Physics*, Version française de G. Delacote et M. Bloch, Paris : InterEditions, 1979.

[4] M. Legrand, *Différents types de modélisation dans l'enseignement*, Recueil des contributions présentées à la séance du Comité Scientifique des IREM, 2003, pp 34-35.

[5] N. Nguyen Thi & A. Birebent,*Didactic research on modeling of periodic phenomena*, Journal of Science, University of Education of Ho Chi Minh City, No. 27 (61), p. 30-40, 2011.

[6] N. Nguyen Thi, *Fonctions trigonométriques et phénomènes périodiques : un accès à la modélisation dans l'enseignement secondaire ?*, Petit x, n0 91, p. 27-48, 2013.

**Textbook cited**

Mathématique 1$^{re}$ S, Collection Déclic, Editions Hachette, 2005

Physique-Chimie 3$^{e}$, Collection Parisi, Editions Belin, 2008

Algebra and Analysis 11, 2007, Educational Publishers.

Physics 12, 2007, Educational Publishers.