Southeast-Asian J. of Sciences Vol. 3, No. 1 (2014) pp. 1-12

PERFORMENCE OF FORECASTING MODEL FOR AVERAGE SURFACE AIR TEMPERATURE PREDICTION OVER SOUTHEAST ASIA

Purinchaya Sornmee and **Dusadee Sukawat**

Dept. of Mathematics, Fac. of Science
King Mongkuts Univ. of Tech. Thonburi, Bangkok 10140, Thailand.
C. of Excellence in Math.s, Mahidol Univ., Bangkok 10400, Thailand
e-mail: dusadee.suk@kmutt.ac.th

Abstract

In this study, the problem of prediction of a given time series has been investigated. The monthly experimental data sets of average surface air temperature over Southeast Asia are analyzed using Educational Global Climate Model (EdGCM). An attempt is being made to examine the performance of a forecasting model based on concept of error growing rate measurement namely Lyapunov exponent (LE). Here, LE and Supremum Lyapunov exponent (SLE) have been evaluated for average surface air temperature under climate change condition. From the LE and SLE behavior, it showed a clear decrease in error growing rate of average surface air temperature after 90-years forecast. This results suggest general good performance for surface air temperature prediction method.

1. Introduction

Educational Global Climate Model (EdGCM) is a climate modeling software suite, created by Columbia University and based on Goddard Institute for Space Studies General Circulation Model II, which is computationally efficient enough for use on personal computers [1]. In order to evaluate the program for

Key words: Educational Global Climate Model, Surface Air Temperature, Lyapunov Exponent.

future use, a series of diagnostic tests of climate sensitivity are conducted to examine the performance of EdGCM.

Although the climate model are able to predict over long time, it is difficult to determine the predictability [2]. The predictability of climate forecasts is determined by the projection of uncertainties in both initial conditions and model formulation. Earth's climate is a prototypical chaotic system [3], implying that its evolution is sensitive to the specification of the initial state. Lyapunov exponent (LE) is one of the most widely used tool to determine this sensitivity in a quantitative way [4] and has verified to be the most useful dynamical diagnostic for chaotic systems [3]. A new predictability measurement, Supremum Lyapunov Exponent (SLE) which is based on the LE have been presented by [5]. The SLE is applied to two cases of the Asian northeast monsoon forecast under a global warming scenario by a shallow water model. The results show that the forecasts from slightly difference initial conditions converge after 3-day forecasts. That is, the shallow water model is not suitable for the purpose of climate downscaling.

The aim of this paper is to estimate performance of the Educational Global Climate Model (EdGCM) in terms of predictability. The impact of changing in greenhouse gases such as atmospheric carbon dioxide and nitrous oxide concentration on average surface air temperatures are analyzed. LE and SLE are chosen to analyze predictability of EdGCM for average surface air temperature over South East Asia region.

2. Theory

2.1 Lyapunov Exponent (LE)

The notion of LE (λ) is based on the average rate of exponential separation of two infinitesimally close trajectories in the phase space. This method represents a mean to measures the rate of convergence or divergence of nearby trajectories [6]. Consider two points in a space (Fig. 1), X_0 and $X_0 + d_0$, each of which will generate a trajectory in that space using some equation or system of equations. If one of the trajectory is used as a reference trajectory, then the separation between the two trajectories will be a function of time.

The growth of the difference d_t between the two trajectories over a time period $\Delta t = t_t - t_0$ can be described by $d_t = d_0 e^{\lambda \Delta t}$ where d_0 is called an initial distance. Thus LE is given by [8]

$$\lambda(t) = \frac{1}{\Delta t} \ln\left(\frac{d_t}{d_0}\right). \tag{1}$$

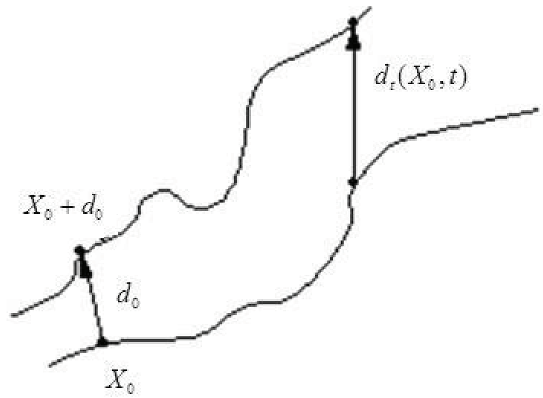

Figure 1: The simple measuring chaos in the sense of LE [7]

For N segments of the nearby trajectories, the average of LE is given by

$$\lambda \equiv \overline{\lambda} = \lim_{N\to\infty} \sum_{j=0}^{N+1} \lambda^{(j)}. \tag{2}$$

2.1 Supremum Lyapunov Exponent (SLE)

The supremum Lyapunov exponent (SLE) provides a measure of the average rate of convergence or divergence of nearby trajectories divergence [5]. The system with more positive exponent indicates sensitive dependence on the initial conditions that is chaotic dynamics and unpredictability. The definition of SLE for predictability measurement is described as follows.

Assume that the evolution of the atmosphere is governed by a nonlinear dynamical system of n-dimension continuous-time, defined by the differential equation,

$$\dot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}(t)) \tag{3}$$

$$\dot{\mathbf{X}}(t) = \frac{d\mathbf{X}(t)}{dt} \tag{4}$$

where $\mathbf{x}(t)$ is the trajectories of the finite-dimensional state space $\mathbb{R}^n$, $\mathbf{x}(t) \in \mathbb{R}^n$ is the state space at time t, that is $\mathbf{x} = [x_1 x_2 x_3 \cdots x_n]^T$ and $\mathbf{F}$ is an n-dimensional vector field. Assume the vector field $\mathbf{F}$ generates the vector in space $\mathbf{x}(t) = f(\mathbf{x}, t)$, such that

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{F}(f(\mathbf{x}, t)) \tag{5}$$

The solution of (5) under the initial condition $\mathbf{x}(0) = \mathbf{x}_0$ is writen as

$$\mathbf{x}(t) = f(\mathbf{X}_0, t) \tag{6}$$

where $f(\mathbf{x}_0(t) : \mathbb{R} \to \mathbb{R}^n$ is the map which describes time evolution of all phase points such that $f(x_0, 0) = x_0$. Let the set $\{f(\mathbf{x}_0, t) : t \in \mathbb{N}\}$ be the trajectory of the system through $\mathbf{x}_0$.

Consider an n-dimensional discrete-time smooth dynamical system of a non-linear model solution that depends only on the initial conditions

$$f(\mathbf{x}, t + \Delta t) = M[f(x_0, t)], t \in \mathbb{N} \tag{7}$$

where $f(\mathbf{x}_0, t) \in \mathbb{R}^n$ is the vector in a state space of the system at the time t, Δt is the time interval, M is the time integration of the numerical scheme from the initial condition $f(\mathbf{x}_0, t)$ to time evolution of the next state $f(\mathbf{x}, t + \Delta t)$.

Define the surface air temperature $x_i(t))$ as the state space of the dynamical system at time t at the point $x_i, i = 1, 2, 3, , N$ (N is the number of grid points in the model domain). The Lyapunov exponent (LE) for $(x_i(t))$ can be written as

$$\lambda = \frac{1}{\Delta t} \ln \frac{||\delta(x_i(t + \Delta t))||}{||\delta(x_i(t))||}, i = 1, 2, \cdots, N \tag{8}$$

Then the characteristic exponents of $(x_i(t))$ are defined by

$$\lambda_{\inf} = \lim_{\Delta t \to \infty} \inf \frac{1}{\Delta t} \ln \frac{||\delta(x_i(t + \Delta t)||}{||\delta(x_i(t))||}, i = 1, 2, \cdots N \tag{9}$$

and

$$\lambda_{\sup} = \lim_{\Delta t \to \infty} sup \frac{1}{\Delta t} \ln \frac{||\delta(x_i(t + \Delta t)||}{||\delta(x_i(t))||}, i = 1, 2, \cdots N \tag{10}$$

If in (9) and (10) the limits exist and

$$\lim_{\Delta t \to \infty} \inf \frac{1}{\Delta t} \ln \frac{||\delta(x_i(t + \Delta t)||}{||\delta(x_i(t))||} = \lim_{\Delta t \to \infty} sup \frac{1}{\Delta t} \ln \frac{||\delta(x_i(t + \Delta t)||}{||\delta(x_i(t))||} i = 1, 2, \cdots N, \tag{11}$$

then the characteristic exponent of $(x_i(t))$ exists and is defined by [5],

$$\Lambda = \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \ln \frac{||\delta(x_i(t + \Delta t))||}{||\delta(x_i(t))||}, i = 1, 2, \cdots, N \tag{12}$$

Therefore, the characteristic exponential of $x_i(t))$ is the rate of divergence or convergence of two nearby trajectories. The supremum of (12) over all of trajectories $x_i(t))$ in the phase space is the point of interest. Define $Z(\mathbf{x}, t)$ as the set of $x \in Z$ for which the limit in (12) exists. Then the supremum Lyapunov exponent (SLE) is defined as in [5],

$$\Lambda_{\sup} = \sup_{x \in Z(\mathbf{x}, t)} \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \ln \frac{||\delta(x_i(t + \Delta t))||}{||\delta(x_i(t))||}, i = 1, 2, \cdots, N \tag{13}$$

If $Z(\mathbf{x}, t)$ is empty then we define $\Lambda_{\sup} = -\infty$.

2.3 Educational Global Climate Model (EdGCM)

The EdGCM is a suite of software that allows users to run a fully functional three dimensions global climate model (GCM) on laptops or desktop computers (Macs and Windows PCs). The heart of a GCM is a model of the Earth's atmosphere which numerically solves five fundamental physical equations as follow [2] The conservation of momentum,

$$\frac{\partial \vec{V}}{\partial t} = -(\vec{V}\cdot\nabla)\vec{V} - \frac{1}{\rho}\nabla p - \vec{g} - 2\vec{\Omega}\times\vec{V} + \nabla\cdot(k_m\nabla\vec{V}) - \vec{F}_d, \tag{14}$$

The conservation of mass,

$$\frac{\partial \rho}{\partial t} = -(\vec{V}\cdot\nabla)\rho - \rho(\nabla\cdot\vec{V}). \tag{15}$$

The conservation of energy,

$$\rho c_p\frac{\partial T}{\partial t} = \rho c_p(\vec{V}\cdot\nabla)T - \nabla\cdot\vec{R} + \nabla\cdot(k_\tau\nabla T) + C + S. \tag{16}$$

The conservation of moisture,

$$\frac{\partial q}{\partial t} = -(\vec{V}\cdot\nabla)q + \nabla\cdot(k_q\nabla q) + S_q + E. \tag{17}$$

The ideal gas law,

$$p = \rho R_d T. \tag{18}$$

where, V - wind velocity, R - radiation vector, C - conductive heating, p - pressure, c_p - heat capacity at const. p, E - evaporation, q - specific humidity, S_q - phase-change source, Ω - rotation of the earth, k - diffusion, F_d - drag force of the earth coefficients, and R_d - dry air gas constant.

EdGCM is a model of the Earth's atmosphere which created for running 143-year predictions from the year 1958 to 2100 [2]. The EdGCM's model grid has 7,776 grid cells which is corresponding to $8^0 \times 10^0$ latitude by longitude in horizontal column and containing 9 vertical layers in the atmosphere. The data required for EdGCM are consisted of boundary conditions, initial conditions, climate forcings, climate feedbacks and climate sensitivity. The climate forcings which impact the results of the run more dramatically, such as greenhouse gases are of interested due to capability to vary its values at the beginning of experiment.

3. Computational details

3.1 Study Domain

The area of interest covers Southeast Asian region between latitude 4^0 S to 28^0 N and longitude 95^0 E to 115^0 E as shown in Figure 2.

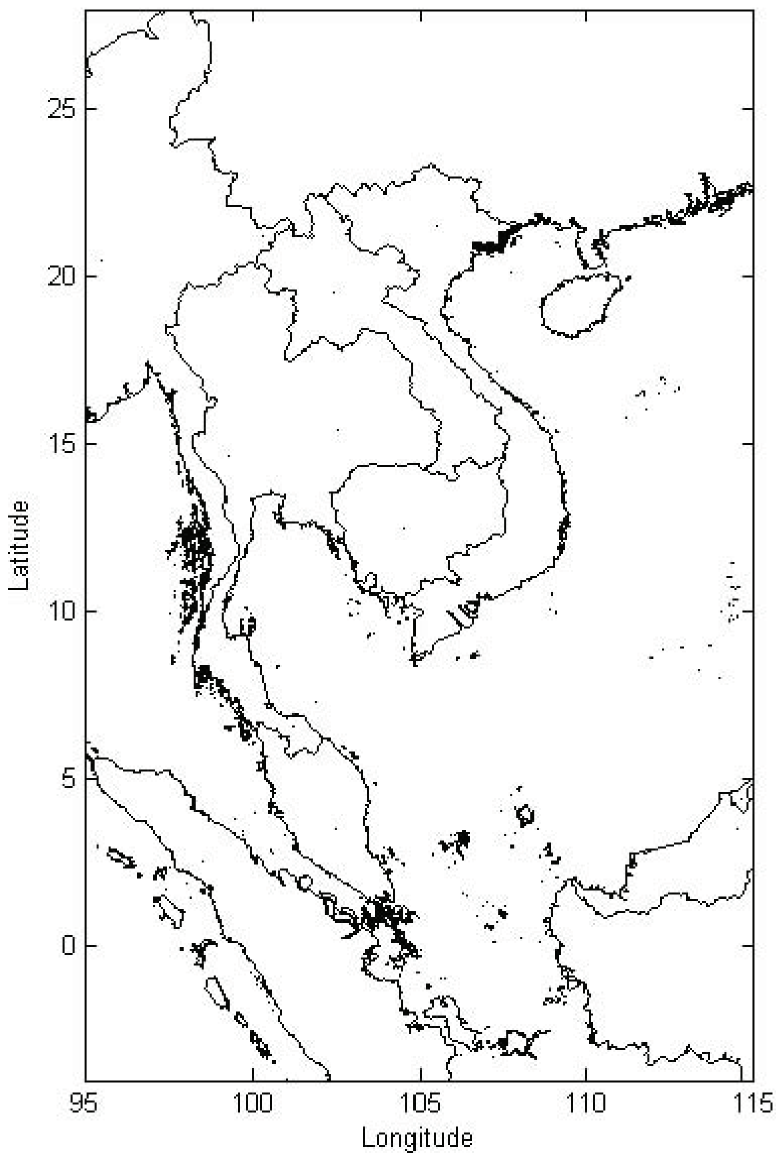

Figure 2: The study domain.

3.2 Experiment Case

The data in this study is the monthly surface air temperature over period of 91 years (1st January 2010 - 31st December 2100) obtained from EdGCM. Summary of experiment designs are shown in Table 1 and 2. Only the data of surface air temperature in April from 2010 to 2100 over Southeast Asia are used for predictability measurement.

Table 1 Control run (CTRL) for EdGCM.

Forcings (Greenhouse gas)	The Control Run (CTRL)
Carbon Dioxide (CO_2)	314.9 ppm
Methane (CH_4)	1.2240 ppm
Nitrous Oxide (N_2O)	0.2908 ppm
CFC-11	0.0076 ppb
CFC-12	0.0296 ppb

Table 2 Perturbation runs for EdGCM.

Forcings (Greenhouse gas)	The Perturbation Runs
Carbon Dioxide (CO_2)	Case1 : 5% increase of CO_2
Methane (CH_4)	Case2 : 5% increase of CH_4
CO_2+CH_4	Case3 : 5% increase of CO_2 + 5% increase of CH_4

From Table 1, in the control run (CTRL) the default values of CO_2 and CH_4 are used as the initial conditions and the model is run for 91-year forecasts from the year 2010 to 2100. The perturbation runs in Table 2 are generated by increasing the amount of CO_2, CH_4 and both of CO_2 and CH_4 to create perturbed runs for Case1, Case2 and Case3 respectively.

4. RESULTS AND DISCUSSION

4.1 Difference in Surface Air Temperature forecasting

The EdGCM is run for 91-year forecasts. The differences of forecast surface air temperatures between the control runs and the corresponding perturbed runs are calculated for each grid point in the study domain. This is done to see sensitivity of the model to perturbations in the initial conditions.

4.1.1 Case1 : 5% increasing of CO2

The difference of the forecast values of surface air temperature between CTRL and Case1 for 91-year forecasts are shown in Figures 3a) - 3d).

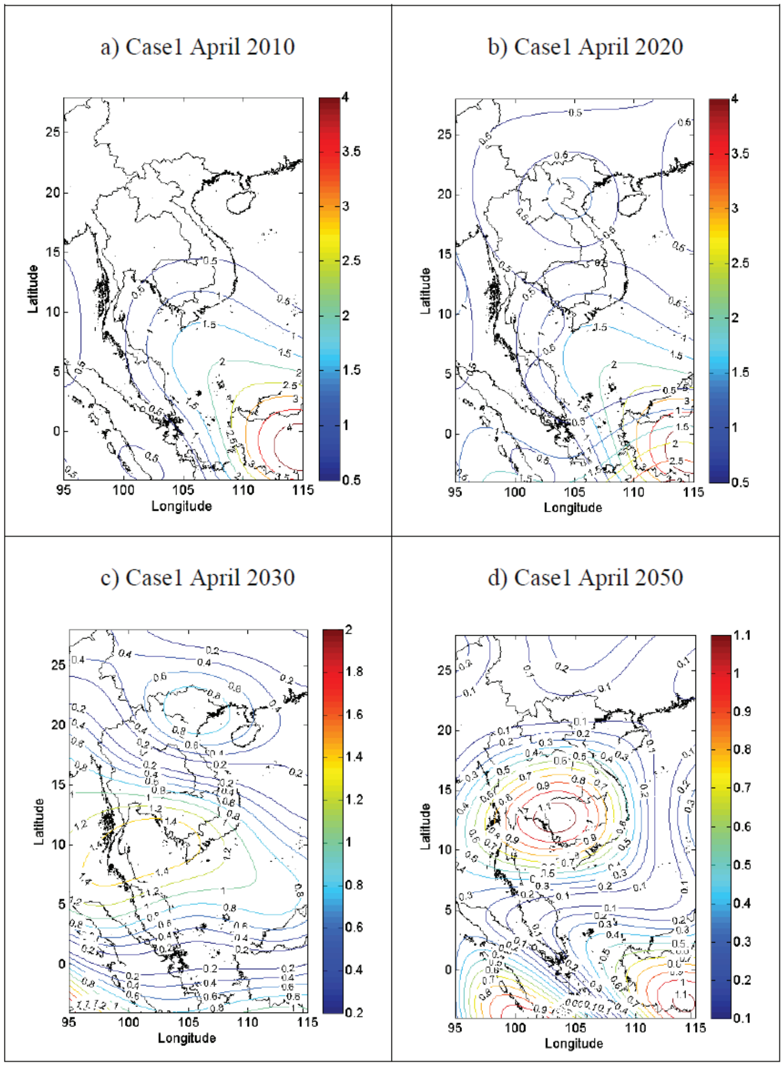

Figure 3. The differences of the forecast values of surface air temperature (°C) between CTRL and Case1 for a) April 2010, b) April 2020, c) April 2080 and d) April 2050.

Figure 3 shows the absolute difference of forecast values of average surface air temperature between CTRL and Case1. It is found that for 50-year forecast the largest difference in average surface air temperature on the study domain is about 4.4^0C. For 60-year, 70-year, 80-year and 90-year forecasts, the largest absolute differences are about 3.0, 2.2, 3.2 and 1.2 ^{0}C, respectively. However, the area with small difference (2 ^{0}C ¡ —difference—) in average surface air temperature expands with forecast period. That is, the forecast average surface air temperature from Case1 converges to that of CTRL after 90-year forecast.

4.1.2 Case2 : 5% Increasing of CH^4

Similarly, Figure 4 shows that the difference of forecast values of average surface air temperature between CTRL and Case2 decreases with forecast period. The values of forecast average surface air temperature from Case2 converges to the forecast from CTRL after 90-year forecast.

Figure 5 shows the difference of forecast values of average surface air temperature between CTRL and Case3 decreases with forecast period. The values of forecast average surface air temperature from Case3 converges to the forecast from CTRL after 90-year forecast.

4.2 Predictability Measurement of EdGCM

The predictability of EdGCM in terms of surface air temperature can be found from computation of LE and SLE for the perturbed runs compare with the control run using Equations (1) and (13). Figure 6 shows the predictability values from LE and SLE of EdGCM for Case1, Case2 and Case3 over study domain.

In Figure 6, it can be seen that at any time, almost all values of LE and SLE for all cases are positive and it decrease monotonously. Figure 6a) shows that from 50-year to 90-year forecasts of all cases have been decreased dramatically. The results also clearly show that LE value from 100-year forecast does not begin to level off until approximately 140-year forecast It is quite possible that by 140-year forecast the LE value is reaching equilibrium rather than decreasing. Similarly, Figure 6b) shows that from 50-year to 100-year forecasts of all cases have been decreased dramatically. The results also clearly show that LE value from 100-year forecast does not begin to level off until approximately 140-year forecast. It is quite possible that by 140-year forecast the LE value is reaching equilibrium rather than decreasing. Thus the predictability of EdGCM forecasting is about 100 years which means that it is a good model for climate forecasting

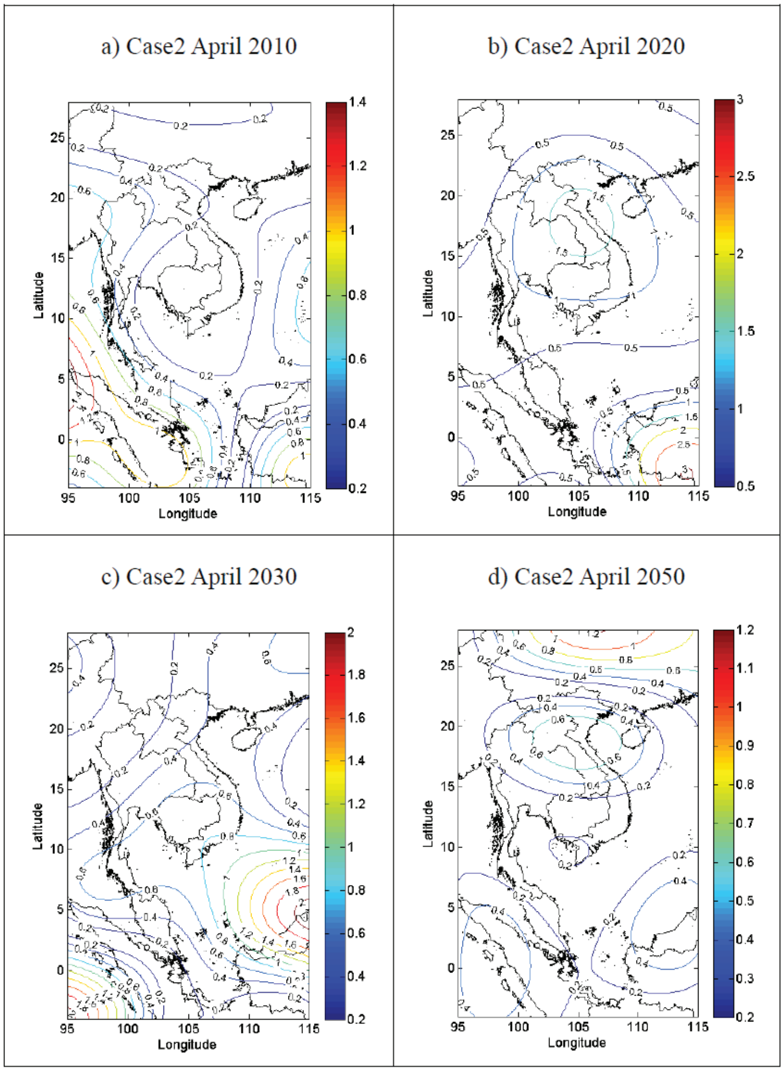

Figure 4. The differences of the forecast values of surface air temperature (°C) between CTRL and Case2 for a) April 2010, b) April 2020, c) April 2030 and d) April 2050.

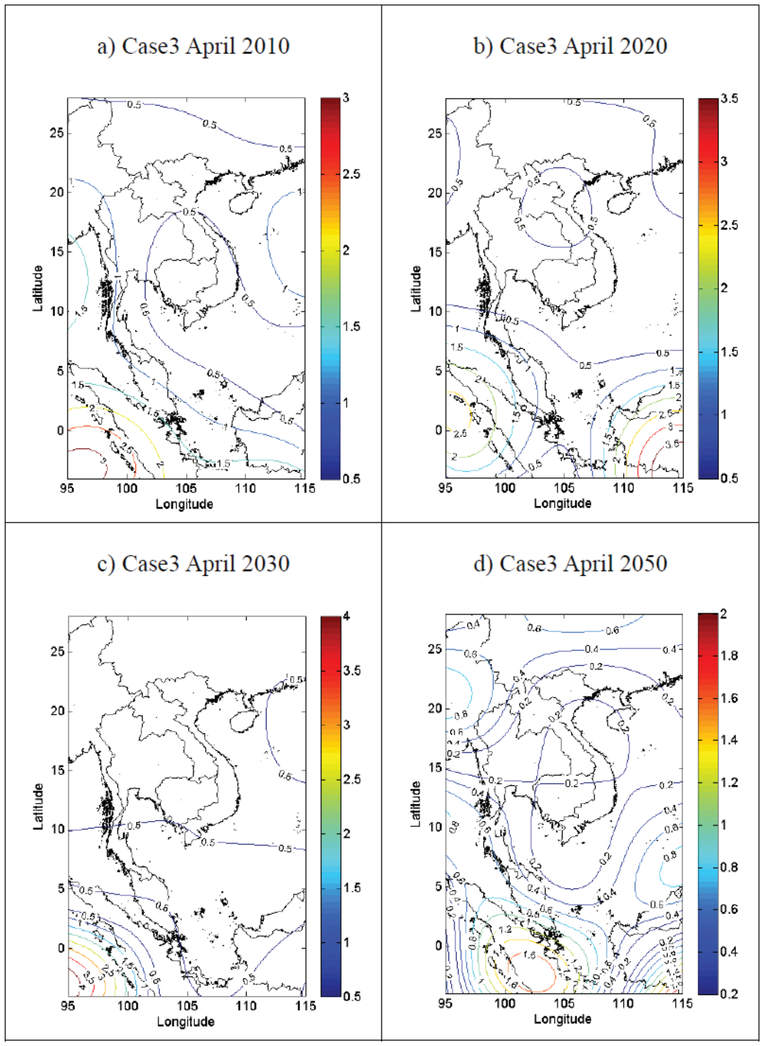

Figure 5. The difference of the forecast values of surface air temperature (°C) between CTRL and Case 1 for a) April 2010, b) April 2020, c) April 2080 and d) April 2050.

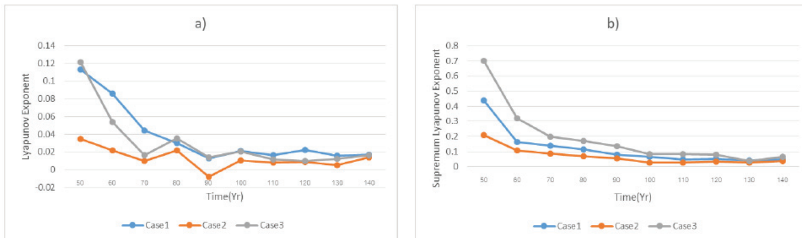

Figure 6. Time evolution for Case1, Case2 and Case3 of a) LE and b) SLE.

5. CONCLUSION

The monthly average surface air temperature prediction from EdGCM is examined with Lyapunov analysis approach. Lyapunov exponent (LE) and Supremum Lyapunov exponent (SLE), as the analysis methods, are used for estimating the growth of small initial errors to determine predictability of EdGCM. The results suggest good performance of EdGCM as it has at least 90 years forecasting predictability. The cases of both 5% increase of CO_2 and 5% increase of CH_4 shows the lowest predictability. To explore the predictability of different regions by EdGCM is an interesting point to be studied in further work.

Acknowledgments This work was supported by Centre of Excellence in Mathematics (CEM) scholarship. Authors would like to thank Department of Mathematics, King Mongkut's University of Technology Thonburi for the research facilities.

References

[1] Mohan, A., and Pecora, K., *Modeling Climate Change and Testing EdGCM Effectiveness,* Thesis, Dickinson College, USA, 2011, 5-8.

[2] M. Chandler,L. Sohl, and A. Marti, EdGCM, "Climate Modeling Workshop", 2006, 11-13.

[3] T.N., Palmer, *Predicting uncertainty in forecasts of weather and climate,* Report Progress in Physics, **63**(2), (2000), 71.

[4] R. Uma and S.R. Selvaraj, *Lyapunov Exponents and Predictability of Cyclonic Disturbances over the North Indian Ocean* Intl. J. of Current Research, **4**(10)(2012), 165-168.

[5] S. Saiuparad and D. Sukawat, *Predictability of the Northeast Monsoon Forecasts by a Shallow Water Model as Measured by the Supremum Lyapunov Exponent,* Far- East J. of Dynamical Systems, **65**(1)(2012), pp. 51-71.

[6] L. McCue, *Applications of Finite-Time Lyapunov Exponents to the Study of Capsize in Beam Seas,* Proceeding of the 8th Intl. Ship Stability Workshop, 6-7 October 2005, Istanbul, pp. 1-9.

[7] G. Elert, The Chaos Hypertextbook [Online], Available : hypertextbook.com/chaos/43.shtml [2013, May 16].

[8] A. M. Marc, *Notes on How to Numerically Calculate the Maximum Lyapunov Exponent,* Orbit, An Intl. Journal, **57**(9)(1995), 2-3.