

# CLASSES OF FUNCTIONS FOR PROPAGATION OF TOPIC TRUST IN SOCIAL NETWORK

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## Abstract

The purpose of this paper is to study functions for computing topic trust in social networks. The focus is to describe classes of functions for propagation of trust among connected peers. Some of them represent properties of relationships between user's interests on topic and trust trustworthiness. Some other functions describe computational methods of topic trust values along a path and from various paths. Based on these classes of functions, we construct an algorithm for computing topic trust via propagation of topic trust among nodes.

## 1 Introduction

There are several models of trust computation of social networks in literature [1][4][6][7], whose approaches are based on interaction among partners or on semantics of contents from posted messages. In the recent work [6], we proposed a model of trust estimation based on interaction which is constructed from two operators concatenation and aggregation. This paper considers a more general approach of estimating topic trust values from a functional approach. The topic worthiness is a function of experience trust and user's topic interest, whereas

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the propagation of topic trust values is computed as a function of connection paths. It is considered as a complementary part of our previous study for a trust computing model in social network [6].

Our purpose is first to investigate properties of topic trust values via itself experience and propagation along various paths. And then we propose classes of functions for estimating topic trustworthiness based on experience evaluation and their combination from various paths.

The remainder of this paper is constructed as follows. Section 2 presents notations and basic concepts. Sections 3 and 4 are devoted to describing classes of functions for experience topic trust and propagation of trustworthiness along paths. Section 5 is conclusion.

## 2 Notations and Basic Concepts

In this paper, an *entry* is named for a comment, a tag, etc., which is a brief piece of information dispatched from some user  $u_i$  to make a description or post information/idea/opinions on an item such as a paper, a book, a film, a thing and so on. Assume that when a user is interested in some topic  $t$ , he is willing to dispatch entries on it. These entries may be classified into classes with respect to various topics. Several techniques have been proposed for such a classification [10][4][2][12][15]. Some necessary notations and concepts for the rest of this paper are presented as follows.

- Let  $\mathcal{U} = \{u_1, \dots, u_n\}$  be a set of users being called *universe* of users in social media. Each user may be considered as an autonomous entity in the system. Each element of  $\mathcal{U}$  is also called a *peer*. A peer, who posts a message to another one, is called *source peer*; whereas the goal peer is also named *sink peer*;
- Let  $I_{ij}$  be a set of all interactions or connections between  $u_i$  and  $u_j$  and  $|I_{ij}|$  be the number of such interactions. Each interaction between users  $u_i$  and  $u_j$  is a transaction at an instant time, which occurs when  $u_i$  posts on wall or to  $u_j$  a message such as comment, like, opinions etc. Denote  $I_{i*}$  to be a set of all users with whom user  $i$  interacts.
- Let  $\mathcal{T} = \{t_1, \dots, t_n\}$  be a set of topics and denote *classifier* (*Entries*, *Topics*) to be the function for classifying entries into classes.

**Definition 1.** Suppose that  $n_k^t$  is the number of entries a user  $u_k$  has dispatched in some topic  $t$ . Then the interest degree of  $u_i$  on topic  $t$  is defined by the

following formula

$$interest_{topic}(i, t) = \frac{1}{2} \left( \frac{n_i^t}{\sum_{l \in \mathcal{T}} n_i^l} + \frac{n_i^t}{\sum_{u_k \in \mathcal{U}} n_k^t} \right) \quad (1)$$

Based on interaction among peers, we can define trust degree among users that is named *experience trust* as follows.

**Definition 2.** *Experience trust of user  $u_i$  on user  $u_j$ , denoted  $trust^{exp}(i, j)$ , is defined by the formula*

$$trust^{exp}(i, j) = \frac{\|I_{ij}\|}{\sum_{k=1}^n \|I_{ik}\|} \quad (2)$$

$\|I_{ik}\|$  is the number of connections  $u_i$  has performed with each  $u_k$ .

The problem is how to compute topic trustworthiness a source peer may rely on some sink peer in both cases when there is direct interaction and via propagation among various peers. The next sections are devoted to presenting a definition and proposing classes of functions for such a computation.

### 3 Classes of Functions for Experience Topic Trust

#### 3.1 Definitions

**Definition 3.** *A topic trust of a source peer  $u_i$  on a sink peer  $u_j$  of  $t$  is a function  $trust_{topic} : \mathcal{U} \times \mathcal{U} \times \mathcal{T} \rightarrow [0, 1]$ , in which  $[0, 1]$  is an unit interval of the real numbers. The value  $trust_{topic}(i, j, t) = u_{ij}$  means that  $u_i$  (truster) trusts  $u_j$  (trustee) of topic  $t$  with respect to the degree  $u_{ij}$ .*

Note that the trust value  $u_{ij}$  depends on both on interest degree on topic  $t$  being obtained from  $j$  defined in (1) and experience trust degree on  $j$  computed via (2). It means that the topic trustworthiness values are defined via a function of two variables: interest degree and experience trust.

We now proceed to define classes of functions named *experience topic trust function* or briefly *expeto function*. Note that Definition 3 expresses an implicit tuition that: (i) the more a peer relies on an opponent, the higher trustworthiness on some topic is; (ii) the higher interest degree of a peer on a topic  $t$  is, the more trust on him it should be assigned. Thus, an expeto function must be monotonic w.r.t. two variables. We have the following definition.

**Definition 4.** *A function  $f : [0, 1] \times [0, 1] \rightarrow [0, 1]$  is an experience topic trust function or expeto one iff it is monotone w.r.t. each variable.*

It is easy to prove the following proposition.

**Proposition 1.** *The following functions are expeto ones:*

- (i)  $f : [0, 1] \times [0, 1] \rightarrow [0, 1]$  is defined by the formula  $f(x, y) = x \times y$ , where  $\times$  is the usual multiplication;
- (ii)  $f : [0, 1] \times [0, 1] \rightarrow [0, 1]$  is defined by the formula  $f(x, y) = e^{x \times y}$ , where  $e^{x \times y}$  is the usual exponential function;

Based on the class of expeto functions, we have the following definition of experience trust.

**Definition 5.** *Suppose that  $trust^{exp}(i, j)$  is the experience trust of  $u_i$  on  $u_j$  and  $interest_{topic}(j, t)$  is the interest degree of  $u_j$  on the topic  $t$ . Then the experience topic trust of  $u_i$  on  $u_j$  of topic  $t$  is defined by the following formula:*

$$trust_{topic}^{exp}(i, j, t) = f_{expeto}(u_{ij}^{exp}, \alpha_{jt}) \quad (3)$$

where  $u_{ij}^{exp} = trust^{exp}(i, j)$ ,  $\alpha_{jt} = interest_{topic}(j, t)$  and  $f_{expeto} : [0, 1] \times [0, 1] \rightarrow [0, 1]$  is an expeto function.

## 4 Functions for Computing Propagation of Topic Trust

### 4.1 Hierarchy of Users for Topic Trust

Our problem is how to estimate a topic trust value in the case there is no any direct interaction between truster  $u_i$  and trustee  $u_j$  but there exists a path  $p(i, j)$  connecting  $u_i$  and  $u_j$ .

There is then a sequence of peers  $u_k$  ( $k = 1, \dots, n$ ), which have connection in couple with each others:  $u_i$  connects with  $u_1$ ,  $u_1$  connects with  $u_2, \dots, u_n$  connects with  $u_j$ . Let  $\Phi(i, j)$  be a set of all paths  $p(i, j)$  connecting  $u_i$  and  $u_j$ . The topic trust estimation is computed by means of middle trustees that have direct interaction with each other and defined via paths from truster  $u_i$  to trustee  $u_j$ . We observe that nodes connecting with a given node may be classified into various levels, which support trust estimation.

**Definition 6.** *Given peers  $u_i$  and  $u_j$ , a propagation path connecting  $u_i$  and  $u_j$  is a sequence of peers  $u_k$  ( $k = 1, \dots, n$ ) such that  $u_i$  connects with  $u_1, u_1$  connects with  $u_2, \dots, u_n$  connects with  $u_j$ .*

**Definition 7.** *Given a user  $u_i$ . A user  $u_j$  is 1-level neighbor of  $u_i$  or 1-neighbor if there is some direct interaction from  $u_i$  to  $u_j$ .*

We make convention that that 0-level of  $u_i$  is  $u_i$ . The concept of  $k$ -level neighbor of  $u_i$  is defined recursively as follows.

**Definition 8.** *Given a user  $u_i$ . A user  $u_j$  is a  $k$ -level neighbor or  $k$ -neighbor of  $u_i$  ( $k \geq 2$ ), if two following conditions are satisfied:*

- (i)  $u_j$  has no direct interaction from any user of  $l$ -neighbor of  $u_i$ , for all  $l \leq k - 1$
- (ii) There is at least a peer of  $(k-1)$ -neighbor of  $u_i$ , which has some direct connection with  $u_j$ .

Denote  $L_i^k$  for all  $k \geq 1$  to be a set of  $k$ -neighbors of  $u_i$ . It is easy to prove the following proposition.

**Proposition 2.** *Given a source peer  $u_i$ . Then there is a number  $n_i$  such that  $L_i^1 \dots, L_i^{n_i}$  are  $k$ -neighbors of  $u_i$  and satisfy the following conditions:*

- (i) For every  $v \in L_i^k$  ( $k = 2, \dots, n_i$ ),  $v$  not being connected with any one in  $\cup_{l=0}^{k-1} L_i^l$ .
- (ii)  $L_i^k \cap (\cup_{l=0}^{k-1} L_i^l) = \emptyset$ , for all  $k \geq 1$ .

Thus, we have a taxonomy of neighbors of  $u_i$  and  $L_i^1 \dots, L_i^{n_i}$  is then called *partition* or *taxonomy* of neighbors of  $u_i$ .

## 4.2 Computational Propagation via Hierarchy

Based on the above taxonomy, we may estimate trust values according to various paths with nodes on levels of partition. For simplicity in presentation, we denote  $u_{kl}$  the experience topic trust value of  $u_k$  on  $u_l$ . Thus, each node  $u_k$  corresponds to a vector  $\vec{u}_k = (u_{k1}, \dots, u_{kn})$ , where  $u_{kl}$  is computed with the formula (3) and  $u_{kl} = \infty$  if there is no interaction among  $u_k$  and  $u_l$ .

Note that if topic trust values of  $u_k$  on  $u_l$  and  $u_l$  on  $u_z$  are  $u_{kl}$  and  $u_{lz}$ , respectively, then trust value  $u_{kz}$  of  $u_k$  on  $u_z$  may not be higher than  $u_{lz}$  and  $u_{kl}$ . Now we proceed to construct the class of functions for estimating topic trust via propagation as follows.

**Definition 9.** *Suppose that  $u_k$  ( $k = 0, \dots, m + 1$ ) is a sequence of nodes connecting  $u_i$  and  $u_j$  with convention that  $u_i = u_0$  and  $u_j = u_{m+1}$ . A function*

$$f_{path}^{trust} : [0, 1]^m \rightarrow [0, 1]$$

*is called path trust function, or briefly patrust, iff it satisfies the property  $f_{path}^{trust}(u_{i1}, \dots, u_{mj}) \leq u_{k,k+1}$  for all  $k = 0, \dots, m$*

It is easy to see that

**Proposition 3.** *The following functions are patrust ones:*

- (i)  $f(x_1, \dots, x_n) = \frac{x_1 + \dots + x_n}{n}$
- (ii)  $f(x_1, \dots, x_n) = \ln\left(\frac{x_1 + \dots + x_n}{n}\right)$
- (iii)  $f(x_1, \dots, x_n) = \min(x_1, \dots, x_n)$
- (iv)  $f(x_1, \dots, x_n) = \prod_{i=1, \dots, n} x_i$

**Definition 10.** *Suppose that  $p(i, j)$  is a path with the length  $m$  connecting  $u_i$  and  $u_j$ . Topic trust of  $u_i$  on  $u_j$  along the path is defined by the following formula*

$$\text{trust}_{\text{topic}}^{p(i,j)}(u_i, u_j) = f_{\text{path}}^{\text{trust}}(u_{i1}, \dots, u_{mj}) \quad (4)$$

where  $u_{kl}$  are topic trust values  $u_k$  relies on  $u_l$  and  $f_{\text{path}}^{\text{trust}}(p)$  is a path trust function.

Our problem is then how to compute overall topic trust from a set of paths  $\Phi(i, j)$  connecting  $u_i$  and  $u_j$ . It is possible to follow one of four strategies:

- *Strategy 1:* Take minimum of all topic trust values according to paths;
- *Strategy 2:* Select the shortest path. It is based on the observation that a shorter path is more reliable;
- *Strategy 3:* It selects the most reliable neighbor with the highest trust value for going further;
- *Strategy 4:* Take mean of all paths. It is based on the fact that when there is no furthermore information, the mean is the best.

The following functions are used for estimating topic trust.

**Proposition 4.** *The path in which nodes appear only once in each level is the shortest path.*

We formulate them in the following definition.

**Definition 11.** *A function  $f : [0, 1]^n \rightarrow [0, 1]$  is a reference topic trust one iff it belongs to the following ones:*

- (i)  $f(x_1, \dots, x_n) = \min(x_1, \dots, x_n)$
- (ii)  $f(x_1, \dots, x_n) = f_{\text{path}}^{\text{trust}}(p_i)$ , where  $p_i$  is the shortest path among  $p_1, \dots, p_n$
- (iii)  $f(x_1, \dots, x_n) = \frac{x_1, \dots, x_n}{n}$

Based on paths connecting  $u_i$  and  $u_j$ , it is able to compute topic trust value for this couple by means of the path trust functions. The trust value is then called topic trust based on reference or briefly *reference topic trust* and denoted  $trust_{topic}^{ref}(i, j, t)$ . We have the following formal definition.

**Definition 12.** Suppose that  $\Phi(i, j)$  to be the set of paths  $p(i, j)$  from  $u_i$  to  $u_j$ . Then the reference topic trust of  $u_i$  on  $u_j$  of  $t$  is defined by the following formula:

$$trust_{topic}^{ref}(i, j, t) = f_{p(i,j) \in \Phi(i,j)} trust_{topic}^{p(i,j)}(i, j, t) \quad (5)$$

in which  $trust_{topic}^{p(i,j)}(i, j, t) = f_{path}^{trust}(u_{i1}, \dots, u_{mj})$  is the topic trust of  $i$  on  $j$  along the path  $p(i, j)$ .

Based on types of topic trust functions, it is able to construct an algorithm for computing topic trust via propagation which is given in the **Algorithm 1**.

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**Algorithm 1** Computing Reference Topic Trust of  $u_i$  on  $u_j$  of topic  $t$  via class of functions

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**Input:** The set of topics  $\mathcal{T} = \{t_1, t_2, \dots, t_n\}$  and the set of users  $\mathcal{U} = \{u_1, u_2, \dots, u_m\}$

**Output:** The trust of  $u_i$  on  $u_j$  of topic  $t$ ,  $computeRefTopicTrust_{topic}^{ref}(i, j, t)$ .

- 1:  $u_{kl}^t \leftarrow trust_{topic}^{exp}(k, l, t)$  //Computing experience trust for all nodes according to (3)
  - 2:  $\mathcal{P} \leftarrow constructTaxonomy(i)$  //constructing the set of  $L_i^k$  ( $k = 1, \dots, n$ )
  - 3: Define  $L_i^s$  containing  $u_j \in L_i^s$
  - 4: **for all**  $t$  in  $\mathcal{T}$  **do**
  - 5:   **for all**  $k = 1, \dots, s - 1$  **do**
  - 6:     **for all**  $u_k \in L_i^k$  **do**
  - 7:        $trust_{topic}^{ref}(k - 1, k, t) \leftarrow f_{p(k-1,k)} trust_{topic}^{p(k-1,k)}(k - 1, k, t)$
  - 8:        $trust_{topic}^{ref}(i, j, t) \leftarrow f_{p(i,j) \in \Phi(i,j)} trust_{topic}^{p(i,j)}(i, j, t)$
  - 9:     **end for**
  - 10:   **end for**
  - 11: **end for**
  - 12: **return**  $trust_{topic}^{ref}(i, j, t)$
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## Conclusions

In this paper, we have introduced a functional approach for computing topic trust. Three classes of functions have been proposed: the class of ones for computing experience trust, another one of trust along a path and the last one on

composing paths. Experience trust is for direct interaction and path one is for reference trust computation to deal with the situation lacking of direct interaction among users. There are some open problems in our work. The first one is to develop further functions in class and take an evaluation. Second, whether reference topic trust estimation depends on selecting the various paths or not. The issues need to be investigated furthermore. We are currently performing experimental evaluation and comparing with other models on computing trust in social network. The research results will be presented in our future work.

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