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ANALYTICAL SCHEME TO ANALYTICALLY SOLVE ATMOSPHERIC ADVECTION-DIFFUSION EQUATION WITH MIXED BOUNDARY CONDITION

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Abstract

On the basis of semiempirical equation of turbulent diffusion obtained analytical solution for the field of the contaminant concentration in the surface layer of the atmosphere with power laws according to the vertical profile of wind speed and mixing coefficient. Solutions obtained in the environment with noted to the background concentration, dry deposition of contaminant on the underlying surface, shuttering or evaporation of the contaminant from the underlying surface and the stationary continuous high emissions sources, i.e. with boundary conditions of the third kind of a stationary source. Scattering, transport, and deposition of heavy dust also considered.

1 Introduction

The problem of pollution modeling in the atmosphere is divided into different cases based on its boundary conditions. Boundary conditions in this context are the interaction between pollutants to surface linings, covers the surface of

Key words: Analytical dispersion modeling, Green function method, diffusion, boundary conditions.

the earth or inversion layer surface. Often the boundary conditions at the surface divided into three main types: Dirchlet boundary conditions corresponding to the case of completely absorbed at the surface, Neumann boundary conditions corresponding to the case of complete reflection at the surface, and the mixed boundary conditions corresponding to the case partly absorbed at the surface. In particular, Neumann boundary conditions represent the worst case for the environment and most authors focus on solving this problem [24], [8] , [22]. Also, In addition, the Dirichlet boundary conditions are also interested many authors, and they represent different analytical solutions and analysis [22]. Mixed boundary conditions are closer to reality than the Neumann and Dirichlet boundary conditions, but finding the analytical solution for this case is more difficult. Mixed boundary conditions take into account the absorption of the land expressed by dry deposition. Dry deposition is the absorption of pollutants in the atmosphere at the Earth's surface. The absorption is caused by land, water or plant photosynthesis. Dry deposition process reduces the concentration of pollutants in the air in remote areas downwind of emission sources, while the level of contamination may increase in the near place of emission sources, depending on the type of material deposition [21]. A mathematical model of the spread of pollutants in the air, taking into account the factors of dry deposition is carried out through the calibration equation Gaussian smoke trail [12], [2], [8]. In particular, the diffusion coefficients K_x, K_y, K_z is considered constant. In this way, the authors based on meteorological observation and correction model coefficients accordingly. This approach has weaknesses in the process of refinement meteorological parameters because it is based on experience. One approach to more theoretical modeling spreads - dry deposition is focused on solving equations substance spread in the atmosphere with reflective boundary conditions at the ground surface [7] . In previous research oriented on analysis [23], the approach is to make the wind velocity and diffusion coefficient regardless of altitude. These approaches, although much progress compared with previous studies but still inconvenient for practical implementation. The approach in the study [21] gave a general algorithm for solving advection diffusion with wind velocity and diffusion coefficient tangled follow exponential law. Source located at any position in the region. The approach is based on the previous results on the Green's function, in which the Green's function is constructed for each block according to the different homogeneous boundary conditions .

An important study using Berliand approach made by the authors [14]. The model describes the interactions between contaminants and surface lining. The authors present the solution advection diffusion problem with mixed boundary conditions. Deposion term is represented by dry deposition velocity v_d and function represents deposition surface: $A(x)$.

The authors use the Green function method. As a result, the authors find analytical solutions for the case of emission source in the ground. In this paper, we present the solution to the problem of pollution spreading for the mixed boundary conditions to calculate the concentration of ground contamination with high emissions sources. The results of the analysis were calculated to assess and argue.

2 Atmospheric advection - diffusion equation

Definition 1. *The problem of two-dimensional steady state advection-diffusion equation in is*

$$\begin{aligned} &u\frac{\partial q}{\partial x} + w\frac{\partial q}{\partial z} = \frac{\partial}{\partial z}\left(K_z\frac{\partial q}{\partial z}\right) + s(x,z) \\ &s(x,z) = Q\delta\left(x - x_s\right)\delta\left(z - z_s\right) \end{aligned} \tag{1}$$

where x, z are the Cartesian coordinates in the downwind direction and the vertical(positive up-ward) direction, respectively, $q(x,z)$ is the ambient concentration of the contaminant,$u(z)$ is the wind speed in the x direction at height z; w is the vertical velocity, K_z is the eddy diffusivities in z direction, $s(x,z)$ is the source strength function (mass/vol air-time), Q is the source strength, (x_s, z_s) is the location of the point source, δ is the Dirac delta function [11].

3 The mixed boundary condition

In this paper, we solve the atmospheric advection - diffusion equation with the mixed boundary condition. The mixed boundary condition has been define as follows.

Definition 2. *The mixed boundary condition of the equation* (1) *is*

$$\begin{cases} q(x,0) = Q\delta\left(z - z_s\right) \\ q = q_0 \ \ at\ z \to \infty, x \to \infty \\ K_z\dfrac{\partial q}{\partial z} - v_d q = A\left(x\right) \ \ at\ z = 0 \end{cases} \tag{2}$$

where q_0 is the ground ambient concentration, v_d is the dry deposition velocity, $A(x)$ is the interaction function between pollutants and ground's surface.

4 Analytical solutions

The atmospheric advection - diffusion equation with mixed boundary condition is considered in this section. The wind speed and the eddy diffusivity profile have been specified to obtain the solution of the equation (1). Using the

Berliand's profile [4], the wind speed and the vertical eddy diffusivity can be expressed as power laws function of height, respectively as follows

$$u(z) = az^m, \quad K_z(z) = bz^n \tag{3}$$

where a and b are constants. There are many of researchers [3], [26], [25] [20] [22] using the wind speed and eddy diffusivity in the closed-form of the equation (3) .

4.1 Case 1: $w = 0$

The equation (1) becomes

$$u\frac{\partial q}{\partial x} = \frac{\partial}{\partial z}\left(K_z\frac{\partial q}{\partial z}\right) + s(x,z) \tag{4}$$

Transforming the variables as follows

$$\xi = \frac{x}{L}, \zeta = \left(\frac{a}{bL}\right)^{\nu}\frac{z^{1-n}}{(1-n)^{2\nu}}, \chi = q - q_0 \tag{5}$$

where $\nu = \dfrac{1-n}{2+m-n}$, L- the limit of the problem.

The equation (1) becomes

$$\frac{\partial^2\chi}{\partial\zeta^2} - \zeta^{1/\nu-2}\frac{\partial\chi}{\partial\xi} = -\zeta^{1/\nu-2}f(\xi,\zeta) \tag{6}$$

and the boundary condition becomes

$$\chi = 0 \text{ at } \xi = 0,\ \zeta \to \infty \tag{7}$$

$$\frac{\partial\chi}{\partial\zeta} - \alpha\chi = \alpha\left[q_0 - \frac{A(\xi)}{v_d}\right] \text{ at } \zeta = 0 \tag{8}$$

where

$$f(\xi,\zeta) = \frac{a}{L}\left[\left(\frac{bL}{a}\right)^{\nu}(1-n)^{2\nu}Z\right]^{-n/(1-n)}s(\xi,\zeta) \tag{9}$$

$$\alpha = \frac{v_d}{b}\left(\frac{bL}{a}\right)^{\nu}(1-n)^{2\nu-1} \tag{10}$$

The solution of the problem (6)-(7)-(8) can be expressed as

$$\chi(\xi,\zeta) = \chi_Q(\xi,\zeta) + \chi_A(\xi,\zeta) \tag{11}$$

where, $\chi_Q(\xi,\zeta)$ is the solution of the equation (6) with the simply form of the condition equation (8) as follows

$$\frac{\partial\chi}{\partial\zeta} = -\Phi(\xi) \tag{12}$$

and $\Phi(\xi)$ is the solution of the homogeneous form of equation (8)

$$\frac{\partial\chi}{\partial\zeta} - \alpha\chi = 0 \qquad at \quad \zeta = 0, \tag{13}$$

$\chi_A(\xi,\zeta)$ is the solution with the Green function $G(\xi-\xi_0,\zeta)$ of the problem with homogeneous boundary conditions Using the Greens function method [10] [19], [9] the solution of the steady-state atmospheric diffusion with homogeneous boundary condition, χ_Q, can be expressed as

$$\chi_Q(\xi,\zeta) = \int_0^{\xi}\int_0^{\infty} G(\xi,\zeta,\xi_0,\zeta_0)\, g(\xi_0,\zeta_0)\, d\xi_0 d\zeta_0 + \int_0^{\xi} G(\xi,\zeta,\xi_0,\zeta_0=0)\,\Phi(\xi_0)\, d\xi_0 \tag{14}$$

$$g(\xi,\zeta) = \zeta^{1/\nu-2} f(\xi,\zeta) \tag{15}$$

where $G(\xi,\zeta,\xi_0,\zeta_0)$ is the Green's function of the equation (6) with Neumann condition: $K_z\dfrac{\partial\chi}{\partial\zeta} = 0 \quad \text{at} \quad \zeta = 0.$

$$G(\xi,\zeta,\xi_0,\zeta_0) = \frac{\nu(\zeta\zeta_0)^{1/2}}{\xi-\xi_0}\exp\left[-\frac{\nu^2\left(\zeta^{1/\nu}+\zeta_0^{1/\nu}\right)}{\xi-\xi_0}\right] I_{-\nu}\left[\frac{2\nu(\zeta\zeta_0)^{1/(2\nu)}}{\xi-\xi_0}\right] \tag{16}$$

where $I_{-\nu}(z)$- modified Bessel function of the first kind, [6]. and

$$g(\xi,\zeta) = \frac{Q}{Q_0}\delta(\xi-\xi_s)\,\delta(\zeta-\zeta_s), \qquad Q_0 = La\left(\frac{bL}{a}\right)^{1-\nu}(1-n)^{1-2\nu} \tag{17}$$

The solution χ_Q is

$$\begin{aligned}\chi_Q(\xi,\zeta) &= \frac{Q}{Q_0}\frac{\nu(\zeta\zeta_s)^{1/2}}{\xi-\xi_s}\exp\left[-\frac{\nu^2\left(\zeta^{1/\nu}+\zeta_s^{1/\nu}\right)}{\xi-\xi_s}\right] I_{-\nu}\left[\frac{2\nu(\zeta\zeta_s)^{1/(2\nu)}}{\xi-\xi_s}\right] \\ &- \frac{\alpha\nu^{1-2\nu}}{\Gamma(1-\nu)}\int_0^{\xi}\frac{\nu^{1-2\nu}}{(\xi-\xi_0)^{1-\nu}}\exp\left(-\frac{\nu^2\zeta^{1/\nu}}{\xi-\xi_0}\right)\frac{Q}{Q_0}\zeta_s^{\frac{\nu-1}{2\nu}}\exp\left[-\frac{\nu^2{\zeta_s}^{1/\nu}}{2(\xi_0-\xi_s)}\right] \\ &\times \sum_{K=0}^{\infty}(-\gamma)^K(\xi_0-\xi_s)^{\frac{(2K+1)\nu-1}{2}} W_{\frac{1-(2K+1)\nu}{2},\frac{\nu}{2}}\left(\frac{\nu^2{\zeta_s}^{1/\nu}}{\xi_0-\xi_s}\right) d\xi_0\end{aligned} \tag{18}$$

where $W_{K,\mu}(z)$ is Whittaker function, which can be defined as follows

Definition 3. *([6]) For arbitrary real parameters a, b and arbitrary real or complex z , the Kummer's equation is defined as*

$$z\frac{du}{dz} + (b - z)\frac{du}{dz} - au = 0 \tag{19}$$

The complete solution of Kummer equation is

$$u = AM\left(a, b, z\right) + BU\left(a, b, z\right)$$

where $M(a, b, z)$ and $U(a, b, z)$ are independent solutions

$$M(a, b, z) = \sum_{s=0}^{\infty} \frac{(a)_s}{(b)_s s!} z^s = 1 + \frac{a}{b}z + \frac{a(a+1)}{b(b+1)2!}z^2 + ..., \tag{20}$$

$$U(a, b, z) = \frac{\pi}{\sin \pi b}\left[\frac{M(a, b, z)}{\Gamma(1 + a - b)\Gamma(b)} - z^{1-b}\frac{M(1 + a - b, 2 - b, z)}{\Gamma(a)\Gamma(2 - b)}\right] \tag{21}$$

A, B are arbitrary constants.

Definition 4. *([6]) For arbitrary real parameters a, b and arbitrary real or complex z , the Whittaker's equation is defined as*

$$\frac{d^2u}{dz^2} + \left(\frac{1}{4} + \frac{K}{z} + \frac{\frac{1}{4} - \mu^2}{z^2}\right)u = 0 \tag{22}$$

The complete solution of Whittaker equation is

$$u = AM_{K,\mu}\left(z\right) + BW_{K,\mu}\left(z\right)$$

where $M_{K,\mu}(z)$ and $W_{K,\mu}(z)$ are independent solutions

$$M_{K,\mu}\left(z\right) = \exp\left(-\frac{1}{2}z\right)z^{\frac{1}{2}+\mu}M\left(\frac{1}{2} + \mu - K, 1 + 2\mu, z\right), \tag{23}$$

$$W_{K,\mu}\left(z\right) = \exp\left(-\frac{1}{2}z\right)z^{\frac{1}{2}+\mu}U\left(\frac{1}{2} + \mu - K, 1 + 2\mu, z\right) \tag{24}$$

A, B are arbitrary constants.

Notes that the series for γ is a convergent series. Therefore, the first approximation value ($K = 0$) is used for computation. Suppose that the point source located in Oyz, i.e $\xi_s = 0$. The relation of WhittakerW function and Kummer

U function [6] is used to compute the concentration at ground's surface ($\zeta = 0$). The equation (18) becomes

$$\begin{gathered}\chi_Q\left(\xi, \zeta=\xi_s=K=0, \zeta_s\right)=\frac{Q}{Q_0} \frac{\nu^{1-2\nu}}{\Gamma(1-\nu)\, \xi^{1-\nu}} \exp\left(-\frac{\nu^2 {\zeta_s}^{1/\nu}}{\xi}\right) \\ -\alpha \frac{Q}{Q_0} \frac{\nu^{3-5\nu}}{\Gamma^2(1-\nu)} \times \int_0^{\xi}\left(\xi-\xi_0\right)^{\nu-1} {\xi_0}^{\nu-1} \exp\left(-\frac{\nu^2 {\zeta_s}^{1/\nu}}{\xi_0}\right) d\xi_0\end{gathered} \tag{25}$$

Evaluating the integral in equation(25) gives

$$\begin{gathered}\chi_Q\left(\xi, \zeta=\xi_s=K=0, \zeta_s\right)=\frac{Q}{Q_0} \frac{\nu^{1-2\nu}}{\Gamma(1-\nu)\, \xi^{1-\nu}} \exp\left(-\frac{\nu^2 {\zeta_s}^{1/\nu}}{\xi}\right) \\ -\alpha \frac{Q}{Q_0} \frac{\nu^{2-4\nu}\Gamma(\nu)\, \xi^{\frac{3\nu-1}{2}} {\zeta_s}^{\frac{\nu-1}{2\nu}}}{\Gamma^2(1-\nu)} \times \exp\left(-\frac{\nu^2 {\zeta_s}^{1/\nu}}{2\xi}\right) W_{-\frac{3\nu-1}{2},-\frac{\nu}{2}}\left(\frac{\nu^2 {\zeta_s}^{1/\nu}}{\xi}\right)\end{gathered} \tag{26}$$

The solution $\chi_A(\xi, \zeta)$ can be found in form:

$$\chi_A(\xi, \zeta)=\int_0^{\xi} G_Q\left(\xi-\xi_0, \zeta\right) A^*\left(\xi_0\right) d\xi_0 \tag{27}$$

where

$$A^*\left(\xi_0\right)=\alpha\left[q_0-\frac{A\left(\xi_0\right)}{v_d}\right] \tag{28}$$

In simple case, setting $A(\xi) = A_0 = const$. Therefore, equation (27) becomes

$$\begin{gathered}\chi_A\left(\xi, \zeta, A_0\right)=\alpha\left(q_0-\frac{A_0}{v_d}\right) \frac{1}{\Gamma(1-\nu)}\left\{\nu \zeta \Gamma\left(-\nu, \frac{\nu^2 \zeta^{1/\nu}}{\xi}\right)\right. \\ -\alpha \frac{\nu^{2(1-\nu)} \zeta^{\frac{\nu-1}{2\nu}}}{\Gamma(1-\nu)\, \Gamma(1+\nu)} \exp\left(-\frac{\nu^2 \zeta^{1/\nu}}{2\xi}\right) \\ \left.\times \sum_{K=0}^{\infty}(-\gamma)^K \Gamma[(K+1)\nu]\, \Gamma[(K+1)\nu-1]\, \xi^{\frac{(2K-3)\nu-1}{2}} W_{-\frac{(2K-3)\nu-1}{2}, \frac{\nu}{2}}\left(\frac{\nu^2 \zeta^{1/\nu}}{\xi}\right)\right\}\end{gathered} \tag{29}$$

At ground's surface $\zeta \to 0$, using the relation of WhittakerW function and KummerU function [6] in case $1 < 1 + 2\mu = 1 + \nu < 2$, equation (29) becomes

$$\begin{gathered}\chi_A\left(\xi, \zeta=0, A_0\right)=\left(q_0-\frac{A_0}{v_d}\right) \frac{\alpha \nu}{\Gamma(1-\nu)}\left\{\nu^{1-2\nu} \xi^{\nu}-\frac{\alpha \nu^{3(1-\nu)} \Gamma(\nu)}{\Gamma(1-\nu)\, \Gamma(1+\nu)}\right. \\ \left.\sum_{K=0}^{\infty}(-\gamma)^K \frac{\Gamma[(K+1)\nu]\, \Gamma[(K+1)\nu-1]}{\Gamma[(K+2)\nu]} \xi^{(K+2)\nu}\right\}\end{gathered} \tag{30}$$

The first approximation value ($K = 0$) is

$$\chi_A\left(\xi,\zeta=K=0,A_0\right)=\alpha\left(q_0-\frac{A_0}{v_d}\right)\frac{\nu}{\Gamma\left(1-\nu\right)}\left[\nu^{1-2\nu}\xi^{\nu}-\alpha\frac{\nu^{3(1-\nu)}\Gamma^2\left(\nu\right)\Gamma\left(\nu-1\right)}{\Gamma\left(1-\nu\right)\Gamma\left(1+\nu\right)\Gamma\left(2\nu\right)}\xi^{2\nu}\right] \tag{31}$$

Substituting equations (26) and (31) into equation (11) gives the equation of concentration at ground's surface with the height point source as follows

$$\begin{aligned}\chi\left(\xi,\zeta=0,\zeta_s\right)=\frac{Q}{Q_0}\frac{\nu^{1-2\nu}}{\Gamma\left(1-\nu\right)\xi^{1-\nu}}\exp\left(-\frac{\nu^2{\zeta_s}^{1/\nu}}{\xi}\right)\\ -\alpha\frac{Q}{Q_0}\frac{\nu^{2-4\nu}\Gamma\left(\nu\right)\xi^{\frac{3\nu-1}{2}}\zeta_s{}^{\frac{\nu-1}{2\nu}}}{\Gamma^2\left(1-\nu\right)}\exp\left(-\frac{\nu^2{\zeta_s}^{1/\nu}}{2\xi}\right)\times W_{-\frac{3\nu-1}{2},-\frac{\nu}{2}}\left(\frac{\nu^2{\zeta_s}^{1/\nu}}{\xi}\right)\\ +\alpha\left(q_0-\frac{A_0}{v_d}\right)\frac{\nu}{\Gamma\left(1-\nu\right)}\left[\nu^{1-2\nu}\xi^{\nu}-\alpha\frac{\nu^{3(1-\nu)}\Gamma^2\left(\nu\right)\Gamma\left(\nu-1\right)}{\Gamma\left(1-\nu\right)\Gamma\left(1+\nu\right)\Gamma\left(2\nu\right)}\xi^{2\nu}\right]\end{aligned} \tag{32}$$

4.2 Case 2: $w \neq 0$ and $n = 1$

It correspond to the case of atmosphere with inversion layer. The Berliand profile in this case can be expressed as

$$\begin{aligned}u^*(z)=az^{m-w/b}\\ K^*(z)=bz^{1-w/b}\end{aligned} \tag{33}$$

Substituting equation (33) into equation(1) gives

$$u^*(z)\frac{\partial q}{\partial x}=\frac{\partial}{\partial z}K^*(z)\frac{\partial q}{\partial z}+s(x,z) \tag{34}$$

Hence, we rename the variables as follows

$$m^*=m-\frac{w}{b},n^*=1-\frac{w}{b},\nu^*=\frac{1-n^*}{2+m^*-n^*} \tag{35}$$

and

$$\begin{aligned}\xi=\frac{x}{L},\zeta\to\zeta^*=\left(\frac{a}{bL}\right)^{\nu^*}\frac{z^{1-n^*}}{\left(1-\nu^*\right)2\nu^*},\alpha\to\alpha^*=\frac{v_d}{b}\left(\frac{bL}{a}\right)^{\nu^*}(1-n)^{2\nu^*-1}\\ Q_0\to{Q_0}^*=La\left(\frac{bL}{a}\right)^{1-\nu^*}(1-n)^{1-2\nu^*},\gamma\to\gamma^*=\alpha^*\frac{\nu^{1-2\nu^*}\Gamma(\nu^*)}{\Gamma(1-\nu^*)}p^{-\nu^*}\end{aligned} \tag{36}$$

The equation of concentration at ground's surface with the height point source becomes

$$\begin{aligned}\chi_w\left(\xi,\zeta^*=\xi_s=0,\zeta_s^*\right)=\frac{Q}{Q_0^*}\frac{{\nu^*}^{1-2\nu^*}}{\Gamma\left(1-\nu^*\right)\xi^{1-\nu^*}}\exp\left(-\frac{{\nu^*}^2{\zeta_s^*}^{1/\nu^*}}{\xi}\right)\\ -\alpha^*\frac{Q}{Q_0^*}\frac{{\nu^*}^{2-4\nu^*}\Gamma\left(\nu^*\right)\xi^{\frac{3\nu^*-1}{2}}{\zeta_s^*}^{\frac{\nu^*-1}{2\nu^*}}}{\Gamma^2\left(1-\nu^*\right)}\exp\left(-\frac{\nu^2{\zeta_s^*}^{1/\nu^*}}{2\xi}\right)\times W_{-\frac{3\nu^*-1}{2},-\frac{\nu^*}{2}}\left(\frac{{\nu^*}^2{\zeta_s^*}^{1/\nu^*}}{\xi}\right)\\ +\left(q_0-\frac{A_0}{v_d}\right)\frac{\alpha^*\nu^*}{\Gamma\left(1-\nu^*\right)}\left[{\nu^*}^{1-2\nu^*}\xi^{\nu^*}-\frac{\alpha^*{\nu^*}^{3(1-\nu^*)}\Gamma^2\left(\nu^*\right)\Gamma\left(\nu^*-1\right)}{\Gamma\left(1-\nu^*\right)\Gamma\left(1+\nu^*\right)\Gamma\left(2\nu^*\right)}\xi^{2\nu^*}\right]\end{aligned} \tag{37}$$

5 Numerical examples

The model has been tested in 2 case studies:

- Case study 1 ($w = 0$),$m = 0.29$, $n = 0.45$, $a = 1.5ms^{-1}.m^{1-0.29}$, $b = 0.025ms^{-1}.m^{1-0.29}$, $x_s = 0m$, $z_s = 40m$, $z = 0m$, $Q = 10gs^{-1}$, $q_0 = 10^{-4}m^{-2}s^{-1}$, $v_d = 0.0001ms^{-1}$, $A_0 = 1.5.10^{-4}$.
- Case study 2 ($w \neq 0$), $w = 0.1$, ($w = 0$),$m = 0.29$, $n = 0.45$, $a = 1.5ms^{-1}.m^{1-0.29}$, $b = 0.025ms^{-1}.m^{1-0.29}$, $x_s = 0m$, $z_s = 40m$, $z = 0m$, $Q = 10gs^{-1}$, $q_0 = 10^{-4}m^{-2}s^{-1}$, $v_d = 0.0001ms^{-1}$, $A_0 = 1.5.10^{-4}$.

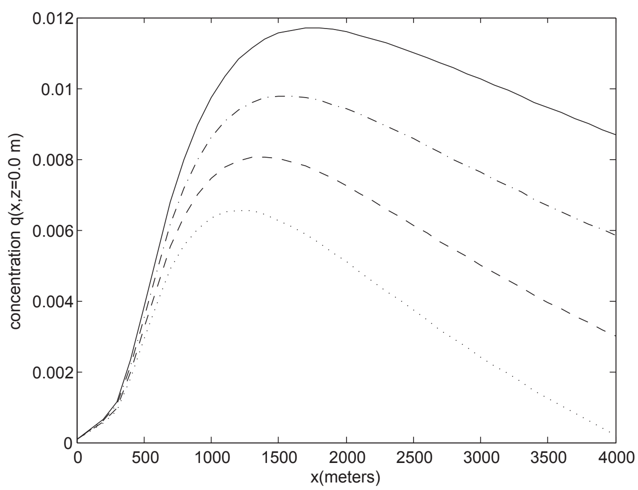

Figure 1: Variation of downwind ground level concentration in case study 1 ($z = 0.0m$) with dry deposition velocities (dotted line: $v_d = 0.025ms^{-1}$; dot-dashed line: $vd = 0.020ms^{-1}$; dashed line: $v_d = 0.015ms^{-1}$; solid line: $v_d = 0.010ms^{-1}$)

On the ground level, the concentration increase first with the distance x from 0, reaches its maximum value, and then decrease with the greater distance x.

6 Conclusion

In conclusion, an analytical solution for the atmospheric advection diffusion equation for point steady source in the third kind of boundary condition is developed. The novelty of this paper is the solution with height point steady

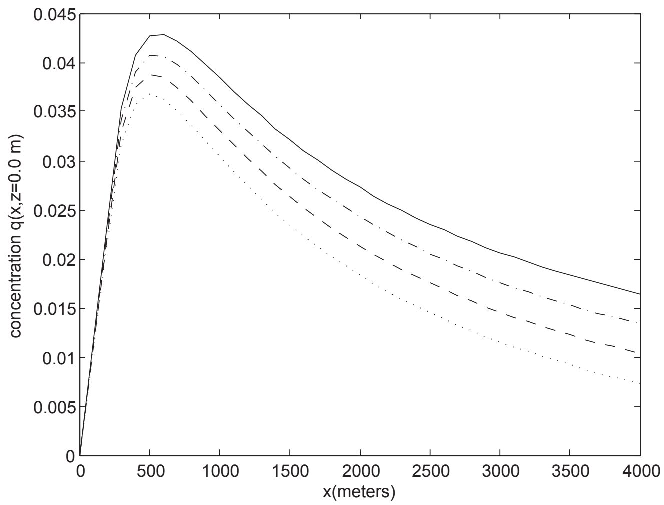

Figure 2: Variation of downwind ground level concentration in case study 2 ($z = 0.0m$) with dry deposition velocities (dotted line: $v_d = 0.25ms^{-1}$; dot-dashed line: $vd = 0.02ms^{-1}$; dashed line: $v_d = 0.015ms^{-1}$; solid line: $v_d = 0.01ms^{-1}$)

source. It closer to reality than the solution with the point source lay in ground [14] .

APPENDICES

Nomenclature

x, z	Downwind distance, height above ground, L
x_s, z_s	The location of the source, L
u, w	The wind speeds along x-axis, y-axis, Lt^{-1}
m	The power-law constant of wind profile, dimensionless
n	The power-law constant of vertical eddy diffusivity profile, dimensionless
a	The diffusivity profile, dimensionless, $L^{1-m}t^{-1}$
b	Parameter in power-law vertical eddy diffusivity profile, $L^{2-n}t^{-1}$
s	The source term in atmospheric advection - diffusion equation, $ML^{-3}t^{-1}$
Q	The emission strength of the point source, Mt^-1
K_y, K_z	The lateral and the vertical eddy diffusivity, L^2t^{-1}
v_d	The dry deposition velocity, Lt^{-1}
$I_v(z)$	The modified Bessel function of first kind, dimensionless
$K_v(z)$	The modified Bessel function of second kind, dimensionless

$W_{K,\nu}(z)$	The WhittakerW function, dimensionless
$U(a,b,z)$	The KummerU function, dimensionless
$\Gamma(z)$	The Gamma function, dimensionless
$\delta(z)$	The delta Dirac function, dimensionless.

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References

[1] Tran Anh Dung, Nguyen Dinh Huy and Bui Ta Long, *Mathematical and computation model to evaluate of surface lining to the extend and scope of the pollution in the atmosphere,* Inter. Conf. on Engineering Mechanics and Automation,Hanoi, Vietnam, (2014), 192-197.

[2] Chamberlain, A. C., "Aspects of travel and deposition of aerosol and vapor clouds", Atomic Energy Research Establishment, Harwell, Berkshire, England, 1953.

[3] Calder, K. L., *Atmospheric diffusion of particulate material, considered as a boundary value problem,* J.Met, **18**(1961), 413-416.

[4] Berliand, M. Y., "Contemporary problems of atmospheric diffusion and pollution of the atmosphere", Translated version by NERC, U.S. EPA, Raleigh, North Carolina, 1975.

[5] S. L. Sobolev and A. J. Lohwater, "Partial Differential Equations of Mathematical Physics", 1964.

[6] Abramowitz, M., and Stegun I. A., "Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables", Dover Publications, New York, 1970.

[7] Seinfeld, J. H., "Atmospheric Chemistry and Physics of Air Pollution", Wiley, New York, 1986.

[8] Turner, D. B., "Workbook of atmospheric dispersion estimates", Environmental Protection Agency (Revised), USEPA No. AP-26 Research Triangle Park, North Carolina, U.S, 1970.

[9] Greenberg, M. D., "Application of Green's Functions in Science and Enoineering", Prentice Hall, Englewood Cliffs, NewJersey, 1971.

[10] Stakgold, I., "Greens Functions and Boundary Value Problems", Wiley, NewYork, 1979.

[11] Dirac, P., "The principles of quantum mechanics", Oxford University Press, Oxford, 1958.

[12] Horst, T. W, *A surface depletion model for deposition from a Gaussian plume,* Atmospheric Environment, **11**(1977), 41-46.

[13] Horst, T. W. and Slinn, W. G. N., *Estimates for pollution profiles above finite areasources,* Atmospheric Environment, **18**(1984), 1339-1346.

[14] Gavrilov, V. P., and Gormalyuk, Y. K., *Dispersion of a poluttant from stationary source in the atmosperic surface layer,* Meteorology and hydrology, **2**(1989), 36-48.

[15] Ermark, D. L., *An analytic model for air air pollutant transport and deposition from a point source,* Atmospheric Environment, **11**(1977), 231-237.

[16] Rao, T. S. and Gabr, M. M., *An analytic model for air air pollutant transport and deposition from a point source,* Journal of Time Series Analysis, **2**(1981), 155-171.

[17] Constantinos V. Chrysikopoulos and Lynn M. Hildemann and Paul V. Roberts, *A three-dimensional steady-state atmospheric dispersion-deposition model for emissions from a ground-level area source,* Atmospheric Environment, **26**(1992), 747-757.

[18] Pasquill, F., *Atmospheric dispersion of pollution,* Quarterly Journal of the Royal Meteorological Society, **97**(1971), 369-395.

[19] Yeh, G. T., *Greens function of a diffusion equation,* Geophysical Research Letters, **2**(1975), 293-296.

[20] van Ulden, A. P., *Simple estimates for vertical diffusion from sources near the ground,* Atmospheric Environment, **12**(1978), 2125-2129.

[21] Lin, J. S. and Hildemann, L. M., *A generalized mathematical scheme to analytically solve the atmospheric diffusion equation with dry deposition,* Atmospheric Environment, **31**(1996), 59-71.

[22] Lin, J. S. and Hildemann, L. M., *Analytical solutions of the atmospheric diffusion equation with multiple source and height-dependent wind speed and eddy diffusivities,* Atmospheric Environment, **30**(1995), 239-254.

[23] Smith, F. B., *The problem of deposition in atmospheric diffusion of the particulate matter,* Journal of the atmospheric sciences, **19**(1962), 429-434.

[24] Huang, C. H., *On solutions of the diffusion deposition equation for point sources in turbulent shear flow,* Journal of Applied Meteorology, **38**(1998), 250-254.

[25] Huang, C. H. and Drake, R. L., *Validation of Gaussian and non-Gaussian diffusion models for a point source,* Applications of Air Pollution Meteorology, (1977), 299-304.

[26] Round, W., *Solutions of the two-dimensional diffusion equations,* Transactions American Geophysical Union, **36**(1955), 395-405.