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A MULTI-ISSUE TRUST MODEL IN MULTIAGENT SYSTEMS: A MATHEMATICAL APPROACH

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Abstract

In the recent years, trust has become a crucial issue in studying agent-based distributed autonomous systems such as Semantic Web or Peer-to-Peer, in which *software agents* need to select the most trustworthy partners to interact. Most current computational trust models are mainly based on two basic factors: personal experience trust and reference trust on a single issue of trust. These models may be not very fruitful when applying to trust systems with multi-issue, in which agents need to infer a trust of some new issue from trusted issues. This status occurs due to lack of information or uncertainty on both experience trust and reference trust of the issue. In this paper, we introduce a trust model that is an extension of the single issue trust one to a multi-issue trust one. Our approach is to investigate a new type of trust - *inference trust*, and then to integrate it into this extension model. The new trust may enable agents to discover his local knowledge about their partners to infer the new trust of their partners on some issue.

1. Introduction

Trust has become a crucial factor in considering agent-based distributed autonomous systems such as Semantic Web or Peer-to-Peer, when a software agent

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needs to select the most trustworthy partner to interact ([1] - [8]). Several proposed models based on their various aspects of trust may be categorized into three main approaches: protocol oriented models, formal models, and computational models. While the two former ones focus on the logic aspects or protocols of trust, the computational models are mainly based on two basic factors: personal experience trust and reference trust (also called reputation).

For instance, in the trust model proposed by Esfandiari and Chandrasekharan [2], two one-on-one trust acquisition mechanisms are proposed. In Sen and Sajja's reputation model [6], both types of direct experiences are considered: direct interaction and observed interaction. In the model proposed by Yu and Singh [9] the information stored by an agent about direct interactions is a set of values that reflect the quality of these interactions. Only the most recent experiences with each concrete partner are considered for the calculations. While the main idea behind the reputation model presented by Carter and Bitting [1] is that "the reputation of an agent is based on the degree of fulfillment of roles ascribed to it by the society". Sabater and Sierra [5] introduced ReGreT, a modular trust and reputation system oriented to complex small/mid-size e-commerce environments where social relations among individuals play an important role. The system takes into account three different sources of information: direct experiences, information from third party agents and social structures. Ramchurn et al. [4] developed a trust model, based on confidence and reputation, and show how it can be concretely applied, using fuzzy sets, to guide agents in evaluating past interactions and in establishing new contracts with one another. Huynh et al. [3] presented FIRE, a trust and reputation model that integrates a number of information sources to produce a comprehensive assessment of an agent's likely performance in open systems. Victor [8] advocates the use of a trust model in which trust scores are (trust, distrust)-couples, drawn from a bilattice that preserves valuable trust provenance information including gradual trust, distrust, ignorance, and inconsistency. Tran and Nguyen [7] introduce a computational model of trust, which is also combination of experience and reference trust and based on fuzzy computational techniques and weighted aggregation operators.

These models are well suitable to applications with a single issue of trust. However, they may not be very fruitful in the context of multi-issue trust system. For example, regarding a system with three trust issues: x, y and z. In general, trust on issue z may be independent from trust on issue x and/or y. But this trust may depend on those on issue x and/or y in a particular group of partners. So, if an agent i needs to know trust about agent j on trust issue z when i has neither experience trust nor reputation about j on issue z, how does i know it? Discovering how to infer trust on issue z from trust on x and/or y is one of potential solutions but it has not been investigated in the current models. The purpose of the paper is to introduce a trust model which is an extension of single issue trust model proposed by Tran and Nguyen [7], to a multi-issue trust model. This model is based on the local knowledge which an agent has collected from their partners on some issue to infer the trust they may assign to the new one.

The remainder of our paper is organized as follows. Section 2 presents the model of trust based only on "standard trust" - experience trust and reputation, for single issue of trust. Section 3 describes the extension of the standard model by adding the inference trust in multi-issue of trust, which is called the extended model of the standard trust. Section 4 will discus about some related aspects of this extension model. The final section is conclusions.

2. Standard Model of Trust

This section is devoted to presenting an overview of the trust model that integrates both reference and inference trusts (more detail, refer to [7]).

Suppose that A $= \{1, 2, ...n\}$ is a set of agents, S $= \{u, v, ...\}$ is a set of trust issues between agents in the system and U is the set of transactions having been done between agent i and agent j on issue u until the current time. Throughout interaction between agents, an agent i may assign a value 0, 0.5, 1 to an agent j, in which 0 is distrust, 0.5 is neutral and 1 is trust.

Agent i's *experience trust* on agent j is the trust value calculated from the trust of all transactions between i and j about issue u in the past. The *reference trust* represents the trust value the agent i has on agent j about issue u based on trust values given by other agents on the agent j. It is calculated from the experience trust of all agents on agent j about issue u that i knows.

The *refex* trust represents the trust that agent i has on agent j about issue u, which is combination of experience trust and reference trust of i on j about issue u. The greater the value has, the more agent i trusts on agent j about issue u.

Definition 1. *Vector $t_{ij} = (t_{ij}^k)$, where $k = 1, ..., \|U\|$ and $i, j = 1, ..., n$, representing trustworthiness of all transactions between agent i and its partner j and ordered by time from latest to oldest, is called a transaction trust vector.*

Definition 2. *Vector $w = (w_1, w_2, ...w_{\|U\|})^T$ is called the vector of all transaction weights if $w_k \in [0, 1],\ k = 1, ..., \|U\|$ are weights of the k^{th} latest transaction based on agent i evaluation such that:*

$$\begin{cases} w_{k_1} \geqslant w_{k_2} \text{ if } k_1 < k_2 \\ \sum_{k=1}^{\|U\|} w_k = 1 \end{cases} \tag{1}$$

Definition 3. *Experience trust of agent i on agent j about issue u is a mapping*

$E^u_{ij} : [0,1]^{\|U\|} \times [0,1]^{\|U\|} \to [0,1]$ *defined by the formula:*

$$E^u_{ij}(t_{ij}, w) = t_{ij} * w \tag{2}$$

where t_{ij} *is the vector of transaction trust of agent* i *on its partner* j *and* w *is the vector of all transaction weights. Then, vector* $E_i = (E^u_{ij}), j = 1, ..., n$ *representing the experience trust of agent* i *on all their partners about* u *is called the experience trust vector.*

Let V be the set of agents which i knows (except j) and $V \subseteq$ A. We call r^u_{ijl} to be the trust of l ($l \in V$) on j about issue u that l shares with i.

Definition 4. *Reference trust* $R^u_{ij} : [0,1]^{\|V\|} \to [0,1]$ *is a mapping (as a non-weighted average) from all experience trusts of agents, who have had transaction with agent* j *and* i *knows, into the reference trust of agent* i *on agent* j*:*

$$R^u_{ij}(r^u_{ijl}) = \begin{cases} \dfrac{\sum\limits_{l \in V} r^u_{ijl}}{\|V\|} & \textit{if } V \neq \varnothing \\ 0 & \textit{otherwise} \end{cases} \tag{3}$$

Resulting from these trust measures, we may construct a definition of combination of two trusts. Intuitively, the combination trust of experience trust and reference trust, denote *Refex trust*, must satisfy the following conditions: (i) It must neither lower than the minimal and nor higher the maximal of experience trust and reference trust; (ii) The more the experience trust is high, the more the refex trust is high; (iii) The more the reference trust is high, the more the refex trust is high. These constraints are defined by the following *combination function*:

Definition 5. *A function* $t : [0,1] \times [0,1] \to [0,1]$ *is called the combination function, denote com-function, if and only if it satisfies the following conditions:*
(i) $min(e, r) \leqslant t(e, r) \leqslant max(e, r)$;
(ii) $t(e_1, r) \leqslant t(e_2, r)$ *if* $e_1 \leqslant e_2$;
(iii) $t(e, r_1) \leqslant t(e, r_2)$ *if* $r_1 \leqslant r_2$.

Definition 6. *Refex trust* T^u_{ij} *of agent* i *on agent* j *about issue* u *is defined by the formula:*

$$T^u_{ij} = t(E^u_{ij}, R^u_{ij})$$

in which t *is a com-function and* E^u_{ij}, R^u_{ij} *are experience and reference trust, respectively.*

3. Extended Trust Model with Multi-issue

This section introduces a multi-issue trust model that is an extension from the standard one presented in Section 2 by adding an inference trust. Intuitively, when an agent i cannot obtain directly a trust value about an issue u from its experience and reference trust, it could infer from the experience and/or reference trust on agents j about all other issues $v \neq u$. The new trust about u that is calculated from trusts about v is called the *inference trust*.

In the whole section, denote $M = \{T_{ij}^{u_1}, T_{ij}^{u_2}, ...T_{ij}^{u_k}\}$ to be the set of well known trusts that agent i has on agent j about the set of issues $\{u_1, u_2...u_k\}$. We will consider how to calculate the trust T_{ij}^u of agent i on agent j about the new issue u based on the given trust set M.

3.1. Inference trust

A candidate inference rule $cg_i^u(M)$ on the set M is defined as follows:

Definition 7. *A function $cg_i^u(M)$ is considered as a candidate inference rule with an proximity ε of agent i on the trust issue u if only if there is at least an agent j such that $T_{ij}^u \in M$ and*

$$cg_i^u(M) - \varepsilon \leq T_{ij}^u \leq cg_i^u(M) + \varepsilon$$

Definition 8. *A support rate of a candidate inference rule $cg_i^u(M)$ is the rate of number of cases the rule satisfies over all possible cases considered by agent i on the trust issue u, i.e.,*

$$\theta_{cg_i^u(M)} = \frac{|\ \{j : T_{ij}^u \in [cg_i^u(M) - \varepsilon, cg_i^u(M) + \varepsilon]\}\ |}{|\ \{j : T_{ij}^u \text{ is known for } i\}\ |}$$

Definition 9. *A candidate inference rule $cg_i^u(M)$ is considered as an inference rule $g_i^u(M)$ with an proximity ε of agent i on the trust about issue u if and only if its support rate $\theta_{cg_i^u(M)}$ no less than an acceptable rate θ_0, i.e.,*

$$g_i^u(M) = cg_i^u(M) \Leftrightarrow (\theta_{cg_i^u(M)} \geq \theta_0)$$

Definition 10. *A minimal independent inference set $M(u,i)$ of trust issue u of agent i is a minimal subset of T, $M(u,i) = \{T_{ij}^{u_1'}, T_{ij}^{u_2'}, ...T_{ij}^{u_k'}\} \subseteq T$, which enables to infer the trust of agent i on any agent j about issue u with a rate $\theta \geq \theta_0$, i.e.,*

$$M = M(u,i) \Leftrightarrow \text{ if } \ (\text{there exists } M' \subseteq M : \theta_{g_i^u(M')} \geq \theta_0) \text{ then } (M' = M)$$

Algorithm 1 depicts how to validate a given set of candidate inference rules on every trust issue u at each agent i: Lines (3-7) are initial of variables, in particular, the set of validated rules and set of minimal independent inference for each rule. For each candidate rule g, it starts with the smallest set of independent set by size of 1 (7), after each loop, the size is increased by 1 in order to have the minimal independent set. In each loop, it counts the number of supports (lines 13-16) and number of considered case (line 17). If the rate of support is greater than the rate threshold, then the rule is validated and it is taken from candidate set and pushed into output set of rules (lines 21-22); the respective minimal independent set is also pushed into output set (line 23). The loop is repeated until the size of independent set is equal that of considered set or there is no more candidate rules to validate (line 28). This algorithm returns the sets of validated rules and their respective minimal independent set.

Definition 11. *The inference trust of agent i on any agent j about the issue u is defined by means of function $g : \mathrm{R}^{|M(u,i)|} \to \mathrm{R}$*

$$I^u_{ij} = g^u_i(M(u,i)) : \theta_{g^u_i(M(u,i))} \geq \theta_0$$

where $M(u,i)$ is the minimal independent inference set of agent i on issue u, $\theta_{g^u_i(M(u,i))}$ is the support rate of rule $g^u_i(M(u,i))$ and θ_0 is acceptable rate for this inference rule.

In case of many rules which infer the trust on the same issue of an agent's partners, the final inference trust on that issue is based on the rule which has the highest rate. Let $G = \{g1^u_i(M_1(u,i)), g2^u_i(M_2(u,i))...gm^u_i(M_m(u,i))\}$ be a set of inference rules which infer trust on issue u of any agent j in the opinion of agent i. Let also $\Theta = \{\theta_1, \theta_2...\theta_m\}$ be the set of support rates of the rules in G. The final inference trust of agent i on the issue u of any agent j is the inference trust from the rule $gk^u_i(M_k(u,i)) \in G$ which has the highest rate $\theta_k \in \Theta$, that is

$$I^u_{ij} = gk^u_i(M_k(u,i)) : \theta_k = Max\{\theta_1, \theta_2...\theta_m\} \tag{4}$$

Algorithm 2 depicts how the inference trust on a trust issue is inferred from well known trust on other trust issues: lines (2-3) are initial for output parameters. For each trust issue at each agent, the algorithm starts by calculating the rate for all possible rules (lines 6-13). Then it finds out the rule with maximal rate (line 14). If it exists, then applying the chosen rule to infer the new inference trust on the current issue (line 16-18). Once a new inference trust is determined, it re-calculates the final trust on the same issue for all related partners (line 19), and then insert this last final trust into set of well known trust (line 20) for the new loop cycle. The process is repeated until there is no more new inference trust inferred (lines 4-25). The output is the set of final trusts about all related partners on all considered trust issues.

3.2. Overall trust

Based on trusts an agent i may obtain from experience, reference and inference trust, we may introduce the *overall trust* whose computation is based on the following principles.

- If the experience trust and reference trust on u is calculated, the overall trust is calculated as in Definition 6. And then, the overall trust must neither lower than the minimal and nor higher the maximal of experience trust and reference trust.
- If experience trust or/and reference trust may not be directly calculated from collected information, then the overall trust is calculated via the inference trust.

A formal definition of overall trust is given based on the combination function given in Definition 5 as follows:

Definition 12. *Suppose that t is a combination function, I^u_{ij} is inference trust and E^u_{ij}, R^u_{ij} are experience and reference trust, respectively. Then the overall trust T^u_{ij} of agent i on agent j about issue u is calculated by the formula:*

$$T^u_{ij} = \begin{cases} t(E^u_{ij}, R^u_{ij}) & \textit{if there exist } E^u_{ij} \textit{ and } R^u_{ij} \\ I^u_{ij} & \textit{otherwise} \end{cases}$$

4. Discussions

In this section, we offer some discussion on the issue related to our extension model of trust. First, does there exists a model of multi-issue trust without inference trust? The answer is yes. For example, the standard model presented in Section . The proposed model is an extension from traditional models of trust which are mainly based on experience trust and reputation with a single issue of trust to one of multi-issue of trust. The inference trust is helpful when agents are lack of information about both experience trust and reputation on the trust issue of any partner. The ability to infer or reason about trust on a new issue, in some point of view, could be considered as a kind of intelligence of agent.

Second, how can we choose or create a set of candidate rules for the validation algorithm 1? This is can be done by either manual selection or by discovering the local knowledge of agents based on some mining algorithm in the literature. The manual selection method has some advantage in case that the candidate rules are intuitive. For instance, in a system of travel service including hotel booking and travel ticket reservation, each agent on behalf of a service provider or client or both of them. Consider the agents that play the role of both service provider and client: they can share the information about trust of other providers. So there is at least three trust issues: trust about hotel booking service (called issue x), trust about ticket reservation service (called issue y), and trust about sharing information of other providers (called issue z). It is intuitive that if trust on issue x is high, then trust on issue y also high, but trust on issue z is not sure because x and y seem to be the same but z is not. So we can create a set of candidate rules for this system as: { (*trust x high* → *trust y also high*), (*trust y high* → *trust x also high*) }, and then define some concrete function for each rule in this set before applying the algorithm 1 to validate them.

5. Conclusion

This paper introduced a trust model, in a mathematical point of view, which is an extension of single issue trust model to a multi-issue trust model based on personal experience trust and reference trust. Moreover, a new kind of trust, inference trust, is also added to this extension model.

Consequently, the model is a combination of three trust factor: personal experience trust, reference trust, and inference trust. On the one side, it enables to estimate the trust on multi-issue compared to single issue of trust in traditional models. In other side, this also enables agents to discover his local knowledge about their partners, to infer the new trust of their partners on some new issue when they lack of information about personal experience trust and reference trust on the issue. Implementing and evaluating the algorithms and applying the extension model to real applications are currently investigated and will be presented in our future work.

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Algorithm 1 Algorithm to validate all inference rules and their minimal independent inference sets

Input: set of current well known trust T_{ij}^{u} of $\forall i$, $\forall j$ and $\forall u$,
set of candidate inference rule G_c for agent i on issue u,
the proximity ε, and the support rate threshold θ_0 for rules

Output: set of inference rules G_{out} of every agent i on each issue u,
set of minimal independent inference sets M_{out} corresponding to inference rule in G_{out}

for all agent i in the system **do**
    for all trust issue u **do**
        $G_c \leftarrow$ set of all candidate inference rules g
        $G_{out} \leftarrow \varnothing$ // set of all inference rules
        $M_{out} \leftarrow \varnothing$ // set of all minimal independent inference sets
        $M^* \leftarrow$ set of all trusts $T_{ij}^{u'}$ which are known by i with $\forall j$, $\forall u'$
        $size \leftarrow 1$ // size of current considered inference sets
        repeat
            for all (subset $M' \subseteq M^*$) **and** ($\mid M' \mid = size$) **do**
                for all candidate rule g in G **do**
                    $N_{support} \leftarrow 0$ // number of supports for rule g
                    $N_{consider} \leftarrow 0$ // number of considered case
                    for all agent j **do**
                        if $T_{ij}^{u} \in [g(M') - \varepsilon, g(M') + \varepsilon]$ **then**
                            $N_{support} \leftarrow N_{support} + 1$
                        end if
                        $N_{consider} \leftarrow N_{consider} + 1$
                    end for
                    $rate \leftarrow N_{support}/N_{consider}$
                    if $rate \geq \theta_0$ **then**
                        $G_{out} \leftarrow G_{out} \cup \{g\}$
                        $G_c \leftarrow G_c \backslash \{g\}$
                        $M_{out} \leftarrow M_{out} \cup M'$
                    end if
                end for
            end for
            $size \leftarrow size + 1$
        until ($size = \mid M^* \mid$) **or** ($G_c = \varnothing$)
    end for
end for
return M_{out} **and** G_{out} for each agent i and each issue u

Algorithm 2 Algorithm to burn all inference trusts

Input: set of current general trust of agent i on agent j about issue u: M
Output: set of new general trust of agent i on agent j about issue u: M'

for all agent i in the system **do**
  $M \leftarrow$ set of all T_{ij}^u well known
  $G \leftarrow$ set of all rule g mined
  repeat
    for all trust issue u **do**
      for all rule g_v in G which infer to trust on issue u **do**
        $M_v(u, i) \leftarrow$ minimal independent inference set of rule g_v
        $\Theta \leftarrow \varnothing$ // the set of rate of respective rule g_v in G
        if $M_v(u, i) \in M$ **then**
          $\theta_v \leftarrow$ the support rate of rule g_v
          $\Theta \leftarrow \Theta \cup \{\theta_v\}$
        end if
      end for
      $\theta_k \leftarrow Max(\Theta)$
      if θ_k exists **then**
        for all agent j in the system **do**
          if T_{ij}^u is not still known **then**
            $I_{ij}^u \leftarrow g_k(M_k(u, i))$
            $T_{ij}^u \leftarrow I_{ij}^u$
            $M \leftarrow M \cup \{T_{ij}^u\}$
          end if
        end for
      end if
    end for
  until there is no new I_{ij}^u inferred
end for
$M' \leftarrow M$
return M'